
Part I: Work and flux forms.

1. 
   (i) Write out the explicit definition of the flux form of a vector field in $\mathbb{R}^2$ given as
   \[ \mathbf{F} = F_1 \mathbf{i} + F_2 \mathbf{j}. \]
   (ii) Find the flux of the field
   \[ \mathbf{F} = x^2 \mathbf{i} + xy \mathbf{j} \]
   through the parabola $y = 2x^2$ oriented so that it joins the points (0,0) to (1,2).

2. Let
   \[ \mathbf{F} = \mathbf{i} + \mathbf{j} \]
   How would you place a directed line segment of length 1 such that the flux across will be maximum? What is the maximal value? How about zero?
   
   \textit{Hint: The quickest solution is by geometric reasoning. Alternatively, you may assume that the segment points in the direction of a vector $\mathbf{v}$ which you must determine, and use a suitable parametrization to compute the flux in terms of $\mathbf{v}$.}

3. Consider the cylinder
   \[ x^2 + y^2 = a^2, 0 \leq z \leq h \]
   oriented by the outward normal. Find the flux of the field
   \[ \mathbf{F} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}. \]

4. Consider $S$ the upper hemisphere
   \[ x^2 + y^2 + z^2 = a^2, z \geq 0 \]
   oriented by the outward pointing normal.
   
   (i) Find the flux of the field
   \[ \mathbf{F} = 3y \mathbf{i} - 3x \mathbf{j} + 2z \mathbf{k} \]
   over $S$.
   
   (ii) Find the flux of the field
   \[ \mathbf{G} = x^2 \mathbf{i} + xy \mathbf{j} + xz \mathbf{j} \]
   over $S$. 
5. Consider \( P \) the paraboloid
\[
z = 9 - x^2 - y^2, \quad z \geq 0
\]
on oriented by the outward pointing normal. Find the outward flux over \( P \) of the field
\[
\mathbf{F} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}.
\]

6. From the textbook, solve the following problems: 6.5.2, 6.5.4, 6.5.18, 6.5.20.

**Part II: Exterior derivative, grad, curl and div.**

7. From the textbook, solve 6.7.2, 6.7.4, 6.7.6.

8. From the textbook, solve 6.8.1, 6.8.9, 6.8.12.

9. For the gravitational field
\[
\mathbf{F} = -GM \cdot \frac{\mathbf{r}}{r^3} = -GM \frac{x}{r^3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]
show that the divergence and curl are both zero:

(i) \( \nabla \cdot \mathbf{F} = 0 \)

(ii) \( \nabla \times \mathbf{F} = 0 \).

Also prove that \( \phi = \frac{GM}{r} \) is a potential for \( \mathbf{F} \) i.e.
\[
\mathbf{F} = \nabla \phi.
\]

*Hint: To organize the calculation neatly, it may be useful to prove first that*
\[
\partial_x r = \frac{x}{r}, \quad \partial_y r = \frac{y}{r}, \quad \partial_z r = \frac{z}{r}.
\]