Math 10C - Fall 2009 - Midterm I

Name: ________________________________

Student ID: __________________________

Section time: _________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 55 points. You have 50 minutes to complete the test.

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Problem 1. [8 points.]

Consider the vectors \( \mathbf{v} = i + 2j, \mathbf{w} = 3i - j. \)

(i) [3 points] Draw the two vectors \( \mathbf{v} \) and \( \mathbf{w} \) in standard position, and draw their sum \( \mathbf{v} + \mathbf{w} \).

(ii) [2 points] Compute the vector \( 2\mathbf{v} + 3\mathbf{w} \).
(iii) [3 points] Find the unit vector \( \mathbf{u} \) in the direction of \( \mathbf{w} \).

Problem 2. [10 points.]

Consider the function
\[
f(x, y) = 1 - (x - 1)^2 - (y - 1)^2.
\]

(i) [3 points] Draw the contour diagram for \( f(x, y) \) and clearly label the level curves. Show the contours for at least three levels.
(ii) [4 points] Draw the graph of $z = f(x, y)$.

(iii) [3 points] Draw the cross sections of $f(x, y)$ with $x = 1$ and $x = 2$. 
Problem 3. [10 points.]

(i) [4 points] Suppose that \( z = f(x, y) \) is a linear function of \( x \) and \( y \), with slope 3 in the \( x \) direction and slope \(-2\) in the \( y \) direction. A change of \(.1\) in \( x \) and \(-.2\) in \( y \) produces what change in \( z \)?

(ii) [6 points] The graph of a linear function \( z = g(x, y) \) passes through the point \((1, 2, 5)\). The graph intersects the \( xz \) plane along the line \( z = 3x + 6 \). Determine the linear function \( g \) and sketch its graph.
Problem 4. [12 points.]

(i) [3 points] Write the following sum in a simpler form
\[
\frac{3}{2} + \frac{3}{2^2} + \frac{3}{2^3} + \ldots + \frac{3}{2^{2009}}.
\]

(ii) [3 points] Find the value of \( z \) for which
\[
1 - z + z^2 - z^3 + \ldots = \frac{3}{4}.
\]
(iii) [3 points] Find the quadratic Taylor polynomial (around 0) for the function $f(x) = \ln(x^2 + 1)$.

(iv) [3 points] The third degree Taylor polynomial of a function $f(x)$ equals

$$P_3(x) = 1 - \frac{x^2}{6} + \frac{x^3}{12}.$$  

Is it true that 0 must be a local maximum of $f$? Justify your answer.
Problem 5. [15 points.]

The outcome $x$ of an experiment has the cumulative density function

$$f(x) = \begin{cases} 
  c \left(1 - \frac{4}{x^2}\right) & \text{for } x \geq 2, \\
  0 & \text{for } x < 2.
\end{cases}$$

(i) [3 points] Sketch the graph of $f(x)$. Show that we must have $c = 1$.

(ii) [3 points] Determine the probability density function.

(iii) [3 points] What is the probability that the outcome of the experiment is between 2 and 4?
(iii) [3 points] What is the median outcome of the experiment?

(iv) [3 points] Find the mean value for the outcome of the experiment.