1. (From A=B p. 104) Consider 
\[ f(n) = \sum_{k=0}^{\lfloor n/3 \rfloor} 2^k \cdot \frac{n}{n-k} \binom{n-k}{2k} \]

(a) Verify that 
\[ (E_n^2 + 1)(E_n - 2)F(n, k) = G(n, k + 1) - G(n, k) \]

where 
\[ G(n, k) = -\frac{2^k \cdot n}{n - 3k + 3} \binom{n-k}{2k-2} \]

(b) Use this to evaluate \( f(n) \). Note that the \( k \) sum is not \(-\infty \leq k \leq \infty\), so extra steps are required.

2. In Koepf’s software we have commands
   - For creative telescoping:
     \[
     \text{sumrecursion}(\text{binomial}(n,k),k,s(n)); \]
     \[
     \text{zeilberger}(\text{binomial}(n,k),k,s(n));
     \]
     \[ -s(n+1) + 2s(n) = 0 \]
     \text{zeilberger} either gives a first order recurrence or fails, while \text{sumrecursion} tries orders 1, 2, 3, …, \text{MAXORDER} (a global variable, default 5).
   - For Sister Celine’s algorithm:
     \[
     \text{fasenmyer}(\text{binomial}(n,k),k,s(n),J); \]
     \text{fasenmyer}(\text{binomial}(n,k),k,s(n),0); fails.
     \text{fasenmyer}(\text{binomial}(n,k),k,s(n),1); gives the same recurrence as above.
   - For doing creative telescoping and solving the resulting recurrence, provided it’s first order:
     \[
     \text{closedform}(\text{binomial}(n,k),k,n); \]
     \[ 2^n \]
     \[
     \text{Closedform}(\text{binomial}(n,k),k,n);
     \]
     \[
     \text{Hyperterm}([1],[],2,n)
     \]
     Use Sister Celine’s algorithm and the Creative Telescoping algorithm to find recurrences for \( s(n) = \sum_k \binom{n}{k}^3 \). The answers are very different. Explain how to reconcile them.

3. Koepf # 7.4, 7.7, 7.11 (Krawtchouk), 7.15, 7.19(d,e), 7.21, 7.24(b).