PATH INTEGRALS ON MANIFOLDS

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Abstract. A typical path integral is a formal expression of the form
\[ \frac{1}{Z} \int f(x)e^{-E(x)}dx \]
where \( \mathcal{F} \) is a space of maps from one manifold to another, \( f \) is a real valued function on \( \mathcal{F} \), \( E(x) \) is the energy of the map \( x \), \( dx \) is "Lebesgue measure" and \( Z \) is a normalization constant. The use of path integrals for "quantizing" classical mechanical systems (whose classical energy is \( E \) started with Feynman in [2] with very early beginnings being traced back to Dirac [1]. Path integrals are still heavily used by physicists for both the quantum mechanics of elementary particles and more recently for conjecturing new topological invariants of manifolds. In this talk, I will discuss joint work with Lars Andersson, on defining the path integral in Eq. (1) when \( \mathcal{F} \) is the space of continuous maps \( (x) \) from \([0,T]\) to a compact Riemannian manifold \((M)\) and \( E(x) \) is the standard Riemannian energy of the path \( x \). The idea is to approximate \( \mathcal{F} \) by finite dimensional subspaces consisting of broken geodesics and then to pass to the limit of finer and finer approximations. This method of defining (1) leads to a quantum mechanical system whose Hamiltonian is of the form \( H = -\frac{1}{2} \Delta + \kappa Scal \), where \( \Delta \) is the Laplacian on \( M \), \( Scal \) is the scalar curvature of \( M \) and \( \kappa \) is a constant which depends on how one interprets \( dx \).

References

[1] P. A. M. Dirac, Physikalische Zeitschrift der Sowjetunion 3 (1933), 64.

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