Homework 1

1. Numerically test the order of accuracy of linear interpolation for finding the zeros, along gridlines, of $\sqrt{x^2 + y^2 + z^2} - 0.5$ in the box $[-1, 1]^3$.

2. Numerically test the order of accuracy of the formula in 2D for enclosed volume of the zero level-set:

$$\int_{[-1,1]^3} h(-\phi) \, dx$$

using

$$h(r) = \begin{cases} 
0, & \text{if } x \leq -3\Delta x \\
(x + 3\Delta x)/(6\Delta x), & \text{if } -3\Delta x < x < 3\Delta x \\
1, & \text{if } x \geq 3\Delta x
\end{cases}$$

for the level-set function $\phi = \sqrt{x^2 + y^2} - 0.5$ in the box $[-1, 1]^2$ using Trapezoidal Rule for the integration.

3. (a) Make a zero level-set in the shape of a dumbbell in 2D.
   (b) Make a zero level-set in the shape of a dumbbell in 3D.

4. Consider the transport equation

$$\phi_t - \phi_x = 0 \text{ in } [-1, 1]$$

with initial condition $u(x, 0) = |x| - 0.5$. Observe the results using Euler’s method in time and the correct 1st order upwind differencing in space when:

(a) the time step $\Delta t = \text{the spatial stepsize } \Delta x$ and periodic boundary conditions are used at the boundary $x = -1, 1$.

(b) the time step $\Delta t = \text{the spatial stepsize } \Delta x/2$ and periodic boundary conditions are used at the boundary $x = -1, 1$.

(c) the time step $\Delta t = \text{the spatial stepsize } \Delta x$ and von Neumann boundary conditions are used at the boundary $x = -1, 1$.

(d) the time step $\Delta t = \text{the spatial stepsize } \Delta x/2$ and von Neumann boundary conditions are used at the boundary $x = -1, 1$.

5. Consider the 2D transport equation

$$\phi_t + v \cdot \nabla \phi = 0 \text{ in } [-1, 1] \times [-1, 1]$$

and choose a unit vector for $v$. Using Euler’s method in time and the correct 1st order upwind differencing in space, move a dumbbell under the velocity field $v$ using the level-set method.