2. (a) Using Theorem 7.6, we have that $x^{(k)} \rightarrow x$ as $k \rightarrow \infty$ in the $\infty$-norm iff $x_i^{(k)} \rightarrow x_i$ as $k \rightarrow \infty$ for each $1 \leq i \leq n$. Here, we have

\[
\begin{align*}
x_1^{(k)} &= 1 \rightarrow 1 \\
x_2^{(k)} &= \frac{1}{k} \rightarrow 0 \\
k_3^{(k)} &= \frac{1}{k^2} \rightarrow 0
\end{align*}
\]

as $k \rightarrow \infty$. Thus,

\[
x^{(k)} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

in the $\infty$-norm.

(b) To see this, notice that for any vector $y$,

\[
0 \leq \|y\|_\infty \leq \|y\|_2.
\]

Now, if $x^{(k)} \rightarrow x$ as $k \rightarrow \infty$ in the $2$-norm, this means that

\[
\|x^{(k)} - x\|_2 \rightarrow 0
\]

as $k \rightarrow \infty$. However, we have

\[
\begin{array}{c}
0 \leq \|x^{(k)} - x\|_\infty \\
\downarrow \\
0 \\
\end{array} \leq \|x^{(k)} - x\|_2
\]

as $k \rightarrow \infty$. Thus, by the squeeze theorem, $\|x^{(k)} - x\|_\infty \rightarrow 0$ as $k \rightarrow \infty$. Thus, $x^{(k)} \rightarrow x$ as $k \rightarrow \infty$ in the $\infty$-norm.
3. Let

\[ A = \begin{bmatrix} 3 & -\sqrt{6} \\ 1 & 2 \end{bmatrix}. \]

(a) \( \|A\|_2 \) is given by

\[ \|A\|_2 = \max_{\|x\|=1} \|Ax\|_2. \]

Now,

\[ \|Ax\|_2 = \sqrt{10(x_1^2 + x_2^2) - (6\sqrt{6} - 4)x_1x_2}. \]

When \( \|x\|_2 = 1 \), this simplifies to

\[ \|Ax\|_2 = \sqrt{10 - (6\sqrt{6} - 4)x_1x_2}. \]

To maximise this expression, since \( 6\sqrt{6} - 4 > 0 \), we would need to minimise \( x_1x_2 \) subject to \( \|x\|_2 = 1 \). That is, minimise \( f(x_1) = x_1\sqrt{1 - x_1^2} \) over the variable \( x_1 \). Using calculus, the minimum will occur where \( f'(x_1) = 0 \). Here, we have

\[ f'(x_1) = (1 - x_1^2)^{\frac{1}{2}} - x_1^2(1 - x_1^2)^{-\frac{1}{2}}. \]

Thus, the extrema are located where

\[ (1 - x_1^2)^{\frac{1}{2}} - x_1^2(1 - x_1^2)^{-\frac{1}{2}} = 0. \]

Multiplying this expression by \( (1 - x_1^2)^{\frac{1}{2}} \), we have

\[ 1 - x_1^2 - x_1^2 = 0, \]

or \( x_1 = \pm \frac{\sqrt{2}}{2} \). To find \( x_2 \), we plug this solution for \( x_1 \) into the constraint equation, \( \sqrt{x_1^2 + x_2^2} = 1 \), and solve for \( x_2 \). Here, we have \( x_2 = \pm \frac{\sqrt{2}}{2} \). Now, to find the correct solution, we wish to have the values of \( x_1 \) and \( x_2 \) which minimise \( x_1x_2 \). Clearly, when they are of opposite sign, this product is smaller than when they are of the same sign, and so, the minimum occurs at \( x_1 = \frac{\sqrt{2}}{2} \) and \( x_2 = -\frac{\sqrt{2}}{2} \). This gives,

\[ \|A\|_2 = \sqrt{10 - (6\sqrt{6} - 4) \left( \frac{\sqrt{2}}{2} \right) \left( -\frac{\sqrt{2}}{2} \right)} \]

\[ = \sqrt{8 + 3\sqrt{6}}. \]
5. Let 

\[ A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \]

The eigenvalues of \( A \) are given by the roots of \( \det(A - \lambda I) = 0 \). Now,

\[
\det(A - \lambda I) = \det \begin{bmatrix} 2 - \lambda & 1 & 0 \\ 1 & 2 - \lambda & 0 \\ 0 & 0 & 3 - \lambda \end{bmatrix} \\
= (3 - \lambda) \det \begin{bmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{bmatrix} \\
= (3 - \lambda)((2 - \lambda)(2 - \lambda) - (1)(1)) \\
= (3 - \lambda)(3 - 4\lambda + \lambda^2) \\
= (3 - \lambda)(3 - \lambda)(1 - \lambda).
\]

Thus, the eigenvalues of \( A \) are 3, 3, and 1.

Now, the eigenvectors associated with the eigenvalue 3 are given by the non-trivial solutions of \((A - 3I)x = 0\). That is,

\[
\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
\]

This is equivalent to \( x_1 = x_2 \) and \( x_3 \) is arbitrary. Thus, any eigenvector associated with the eigenvalue 3 can be written as

\[ x = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

such that either \( \alpha \neq 0 \) or \( \beta \neq 0 \) (or both).

Finally, the eigenvectors associated with the eigenvalue 1 are given by the nontrivial solutions of \((A - I)x = 0\). That is,

\[
\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
\]
This is equivalent to \( x_1 = -x_2 \) and \( x_3 = 0 \). Thus, any eigenvector associated with the eigenvalue 1 can be written as

\[
x = \gamma \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}
\]

such that \( \gamma \neq 0 \).