Homework #6

1. Let

\[ A = \begin{bmatrix} -2 & 2 & 3 \\ 3 & 1 & 4 \\ -2 & -1 & 4 \end{bmatrix}. \]

Using initial guess of eigenvector \( x^{(0)} = [1, -1, 2]^T \), Find \( x^{(2)} \) and \( \lambda^{(2)} \) using the power method.

2. For each part, find the interpolating polynomial for the data points

\((-1, 2), (1, 3), (2, -2)\).

by writing down a linear system involving \( p(-1) = 2, p(1) = 3, p(2) = -2 \) and solving it for the unknown coefficients \( a, b, c \):

(a) \( p(x) = ax^2 + bx + c \).
(b) Lagrange form: \( p(x) = a \frac{(x-1)(x-2)}{6} + b \frac{(x+1)(x-2)}{-2} + c \frac{(x+1)(x-1)}{3} \).
(c) Newton form: \( p(x) = a + b(x + 1) + c(x + 1)(x + 1) \).
(d) Simplify each and show they are all the same polynomial.

3. Write down three different polynomials of any degree that interpolate the data points

\((-1, 2), (2, 3)\).

Verify that they are indeed different polynomials.

4. Write down the Lagrange forms for the interpolating polynomial for the data:

(a) \[
\begin{array}{c|ccc}
  x & -1 & 0 & 1 \\
  f(x) & -2 & 0 & 3 \\
\end{array}
\]

(b) \[
\begin{array}{c|ccc}
  x & -1 & 0 & 1 & 3 \\
  f(x) & -2 & 0 & 3 & 3 \\
\end{array}
\]

5. (a) Write down the divided difference table for the data:

\[
\begin{array}{c|ccc}
  x & -1 & 0 & 1 \\
  f(x) & -2 & 0 & 3 \\
\end{array}
\]

(b) Use your divided difference table to write down the Newton forms for the interpolating polynomials for the data points:

i. \((-1, -2), (0, 0)\).
ii. \((0, 0), (1, 3)\).
iii. \((-1, -2), (0, 0), (1, 3)\)
(c) Modify your divided difference table to add in the data point \((3, 3)\) and use it to write down the Newton form for the new interpolating polynomial.

6. Consider data points \((x_0 - h, f(x_0 - h)), (x_0, f(x_0)), (x_0 + h, f(x_0 + h))\).

(a) Find the interpolating polynomial \(p(x)\) passing through these points in either Lagrange or Newton form.

(b) Show \(p'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}\).

(c) Show \(p''(x_0) = \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}\).

(d) Show \(\int_{x_0 - h}^{x_0 + h} p(x) \, dx = \frac{h}{3}(f(x_0 - h) + 4f(x_0) + f(x_0 + h))\).

7. (Matlab)

(a) Write a Matlab function that inputs \(x\), an array of \(x\)-coordinates of data points; \(y\), an array of \(y\)-coordinates of data points; \(n\), the total number of data points; and \(z\), one location on the \(x\)-axis. Have it output the value of the interpolating polynomial for the data points, computed using Lagrange form, at \(z\). Write out or print out your program.

(b) Use your program to approximate \(f(2)\) for the table of data

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Print out or write out your answer.

8. (Math 274) Prove uniqueness of polynomials of degree \(\leq 3\) satisfying

\[
\begin{array}{c|cc}
  x & x_0 & x_1 \\
  f(x) & y_0 & y_1 \\
  f'(x) & z_0 & z_1 \\
\end{array}
\]

when \(x_0 \neq x_1\), by studying the derivative of \(p(x) - q(x)\), for two such polynomials \(p, q\), and counting the number of roots from the data and from Rolle’s Theorem, and applying Fundamental Theorem of Algebra.