Homework #8

1. Find $b, c, d$ so that the following is a natural cubic spline

$$S(x) = \begin{cases} 
1 + 2x - x^3, & \text{if } 0 \leq x < 1, \\
2 + b(x - 1) + c(x - 1)^2 + d(x - 1)^3, & \text{if } 1 \leq x < 2.
\end{cases}$$

2. (a) Consider the data points

$$(-1, -1.1), (-1, -1), (0, 0.9), (0.5, 1.8), (1, 3.2).$$

Write down the normal equations for linear least squares.

(b) Solve the normal equations to get the best fitting line in the least squares sense.

(c) Write down the normal equations for quadratic least squares. You do not need to solve this linear system.

3. Consider the $n$ data points

$$(0, y_1), (0, y_2), \ldots, (0, y_n).$$

Find the constant function best fitting this data in the least squares sense. What is another name for this constant?

4. (a) Consider the data points

$$(0, 1), (0.1, 1.01), (0.2, 1.04), (0.3, 1.09), (0.4, 1.16).$$

Approximate the first derivatives of $f$ at each node location using the best of either forward, backward, or central differencing.

(b) Find the exact absolute errors at each node location using the fact that the data comes from $f(x) = 1 + x^2$.

(c) Use the error expression for central differencing to explain why its approximations were exact.

(d) Approximate the second derivative of $f$ at $x = 0.2$ using central differencing.

5. (a) Let $f(x) = \sin x$. Approximate $f'(1)$ using central differencing with $h = 0.1, 0.05, 0.025$.

(b) Calculate the exact absolute errors $E(h)$ for each of your approximations.

(c) Calculate $E(h)/E(h/2)$ for each of $h = 0.1, 0.05$. What integer do you think this converges to as $h \rightarrow 0$?

(d) Let $E(x, h)$ be the absolute error of the central differencing approximation of $f'(x)$ using stepsize $h$. Estimate $h$ such that $E(x, h) \leq 10^{-10}$ is satisfied for all $x$.

6. Use Taylor series to show $\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$ as an approximation of $f''(x)$ has absolute error $O(h^2)$. 

1
7. (Matlab) Write a Matlab function that inputs the vector $x$ of $x$-values, $y$ of $y$-values, and integer $n$ and outputs the coefficients $a$ and $b$ of the best fitting line $a + bx$ in the least squares sense for the $n$ data points given by $x$ and $y$.

8. (Math 274) Consider the $n$ data points

$$(0, y_1), (0, y_2), \ldots, (0, y_n)$$

with $y_1 \leq y_2 \leq \ldots \leq y_n$ and $n$ odd. Find the constant $C$ that minimizes

$$\sum_{i=1}^{n} |y_i - C|.$$

What is another name for this constant?