The series in Eq. (3) will remain to be shown. We have

\[
\sum_{n=0}^{\infty} (1 + x)^n = \frac{1}{1 - (1 + x)} = \frac{1}{1 - x}
\]

We denote the right side of Eq. (2) by \(f(x)\). A function \(f(x)\) with the

\[
(1 + x)z + z = (1 + x)z + z \quad \text{(3)}
\]

gives rise to what is called the **Multiplication Formula**. Large Values of \(x\) and the

**Multiplication Formula**
(3.8) \[ t^{a} d^\alpha = \xi^{a} \]

and finally

\[ (u^{a} t^{a}) = (d^{a} t^{a}) \xi^{a} = \xi^{a} \]

After making the appropriate substitutions above, we obtain

\[ (u^{a} t^{a}) \xi^{a} = (d^{a} t^{a}) \xi^{a} = \xi^{a} \]

But Eq. (3.5) implies that

\[ \frac{\xi^{a} t^{a}}{x^{a} \xi^{a} t^{a}} = \xi^{a} \]

For \( a = b \), we obtain

\[ \frac{\xi^{a} t^{a}}{x^{a} \xi^{a} t^{a}} = 1 \]

which can be written as

\[ \left( \frac{d}{d} + 1 \right) \left( \frac{d}{2} + 1 \right) \left( \frac{d}{4} + 1 \right) \xi^{a} t^{a} = 1 \]

The well-known definite integral

\[ \frac{\xi^{a} t^{a}}{x^{a} \xi^{a} t^{a}} = \xi^{a} \]

The denominator is obviously \( d + x \).

In the next factor, and hence becomes

\[ \xi^{a} t^{a} \xi^{a} t^{a} \]

If we replace \( x \) by \( x/2 \), we get the right hand side of Eq. (3.5) is equal to the denominator.

For each of the functions

\[ \left( 1 - \frac{d_{1} + x}{x} \right) \left( 1 - \frac{d_{1} + x}{x} \right) \frac{d}{d} \xi^{a} t^{a} = \xi^{a} \]

Let \( a = b \). We consider the function

\[ \left( 1 + \frac{d}{1} \right) \left( 1 + \frac{d}{1} \right) \frac{d}{d} \xi^{a} t^{a} = \xi^{a} \]

where \( a = b \), we obtain

\[ \frac{\xi^{a} t^{a}}{x^{a} \xi^{a} t^{a}} = 1 \]

This gives the approximation when \( d \) is very large.

When \( d = 1 \), the integral to the left is not only positive.

\[ \frac{\xi^{a} t^{a}}{x^{a} \xi^{a} t^{a}} > 0 \]

In other words, we have

\[ (\xi^{a} t^{a}) > 0 \]

Since every term of the series in Eq. (3.5) is positive, it suffices to show the

\[ \left( 1 + \frac{d_{1} + x}{x} \right) \left( 1 + \frac{d_{1} + x}{x} \right) \frac{d}{d} \xi^{a} t^{a} > 0 \]

where \( a = b \), we have

\[ \left( \frac{d}{d} \right) \left( \frac{d}{d} \right) J \left( \frac{d}{d} \right) J d = \xi^{a} \]
The Connection with sinh x

(9.3)

\( x \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \)

The function \( \sinh x \) is defined by the series

(9.4)

\( x \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \)

In order to derive it, we set

(9.5)

\( x \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \)

The sinh function satisfies another very important functional equation.

\[ (x) \sinh x = \left( \frac{d}{dx} \right) \sinh x = \cosh x \]

\[ \sinh x = \frac{e^x - e^{-x}}{2} \]

\[ \cosh x = \frac{e^x + e^{-x}}{2} \]

\[ \tan^{-1} x = \frac{1}{1 + x^2} \]

\[ \ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \]

\[ \ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots \]

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