Math 280B, Winter 2012

Homework 1

1. Problem 5 (page 261)

2. Problem 7 (page 261)

3. Problem 20 (page 263)

4. For each $n = 1, 2, \ldots$ let $X_n$ be uniformly distributed over the finite set $\{1, 2, \ldots, n\}$. That is,
   \[ P[X_n = k] = \frac{1}{n}, \quad k = 1, 2, \ldots, n. \]

Show that
   \[ \frac{X_n}{n} \xrightarrow{d} U, \quad n \to \infty, \]
   where $U$ has the uniform distribution on $[0, 1]$.

5. Let $X_1, X_2, \ldots$, be iid non-negative random variables with common distribution function $F$. Suppose that $F$ has a density function $f$ (i.e., $F$ is “absolutely continuous”):
   \[ F(x) = \int_0^x f(u) \, du, \quad x \geq 0, \]
   and suppose that
   \[ \lim_{u \to 0^+} f(u) = \lambda > 0. \]

Define $Y_n := \min(X_1, \ldots, X_n)$ for $n = 1, 2, \ldots$. Show that the sequence $\{nY_n\}$ converges in distribution, and find the limit distribution.