Maya Mathematics and Science

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Math 163

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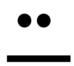
The Maya civilization spread throughout a vast part of Central America. Mayan people lived throughout Southern Mexico, Guatemala, Belize, and Western Honduras. Despite the idea that the ancient Maya "emerged from barbarism," as Morley describes, "during the first or second century of the Christian Era," they were one of the most intelligent civilizations of their time (Morley, p. 2). Despite the fact that they did not have the tools that are considered essential to produce fine pieces of art, sculpture, and architecture, the ancient Maya were great artists and architects. The Maya also made great discoveries in the field of mathematics and astronomy (Ifrah, p. 297). The Maya are considered be great "experts in math, astronomy, astrology and other sciences"(Sipac, p. 21). Compared to other indigenous civilizations in the Americas, the Maya had the most sophisticated numerical system. Their number system is considered to be better than the Aztec system and even the Roman system. The Priests were the scientists and mathematicians of the Maya civilization. They were the "sky watchers, experts in numeration, and experts in the calendar" (Lounsbury, p. 759). Unfortunately, the fact that the majority of the wise men died as a result of epidemics, wars, and colonization hindered any further development of mathematics and science. All that remains of the Maya are inscriptions, such as those in the Dresden Codex, which has allowed scholars to learn about the ancient Mayas numerical system as well as their astronomical and astrological findings. According to Lounsbury, the Maya did not leave "mathematical or astronomical methods or theories. There is of posing of a problem, proof of a theorem, or statement of an algorithm" (Lounsbury, p. 760). However, the Maya's inscriptions have allowed scholars to learn that the Maya used a vigesimal system, one for arithmetic purposes an one for calculating the passage of time, that they developed a very sophisticated calendar, and made discoveries in astronomy that modern scientists could not have been able to do without the aid of technology..

Instead of using a number system with base ten as we use, the ancient Maya used a number system with base 20, also known as a vigesimal system. The Maya dealt with 20 essential digits instead of ten digits as it is done in base ten. In a vigesimal system, the number in the second position is twenty times that of the numeral; the number in the third position is 20^2 times that of the numeral; the number in the fourth position is 20^3 times that of the numeral, e.t.c. The place values were 1s, 20s, 400s, 8,000s, 160,000s, and so on.. In the Mayan language, 20 was called *kal,* 400 was called *bak,* 8000 was called *pic,* 160,000 was called *Calab,* 3,200,000 was called *kinchil,* and 64,000,000 was called *alau* (Lounsbury, p. 762). Just "like our numbering system, they used place values to expand this system and to allow the expression of very large numbers" (Lounsbury, p. 762). For example, to express 352,589, using base ten, we would write $3\times10^5+5\times10^4+2\times10^3+5\times10^2+8\times10+9\times10^0$. The Maya would express this number in a similar fashion except that they would use base 20: $2\times20^4+4\times20^3+1\times20^2+9\times20+9\times20^0$. Moreover, instead of writing out 352,589, the Maya would use a shorthand notation and write it as 2.4.1.9.9, where the numbers 2, 4, 1, 9, and 9 represent the "coefficients" in front of the powers of 20. Thus, using a vigesimal system gave the Maya great advantage because it facilitated the expression of very large numbers and having the feasibility to do this would enable them to count time.

Scholars have found two probable reasons for why the Maya used a vigesimal system. One of the reasons posed is that twenty is the "total number of fingers and toes" (American Scientists, p. 249). They made twenty "their first higher unit because twenty finished a person" (Seidenberg, p. 382). While we use our fingers for counting, the Maya used their fingers and their toes. For example, when referring to the number twenty, they would simply say, "one person." Also, when referring to the number forty, they would say "two people." One of the most important reasons for the use of a vigesimal system was that it was appropriate for making calendrical calculations.

The Maya came up with two symbols, a dot and a bar, to represent numbers, instead of representing them by different symbols as it is done in our number system and that of the Romans. It is believed that because the cocoa bean was used as the "unit of currency" throughout Central America, the Mayas adopted a dot to represent unity or one (Lounsbury, p.762). Another reason to believe that the Maya got the idea of using a dot from the cacao bean, is the fact that they packaged the cacao beans in "quantities of 8000" to a bag (Lounsbury, p. 762). As noted earlier, 8000 is one of the place values of the vigesimal system (20^3). Moreover, the name for 8000 is *pic,* which is also the name for the sack used to pack cacao beans. Thus, the Mayas found it convenient to call 8000 *pic* or *sack* since every sack contained 8000 cacao beans.

Unlike the Aztecs, who also used a vigesimal system and had different symbols for one, twenty, and the powers of twenty, the Mayas used a dot to represent 1, 20, 400, 8000, 160,000, and any further powers of twenty. The second symbol that the Mayas developed was a bar to represent five. However, the bar was always used to represent five and could be repeated up to three times. The numbers one through nineteen would be represented by repeatedly using dots and bars. For example, one was represented by one dot, two by two dots, three by three dots, four by four dots, and, once five was reached, a bar represented it. Thus, dots could be repeated up to four times (Lanczos, p. 17). Six would be represented by a bar and a dot; seven by a bar and two dots; and, once ten was reached, two bars would represent it. If there were multiple bars involved to represent a number, they would be aligned vertically. If there were multiple dots involved, they would be aligned horizontally (See table on the last page for the Mayan symbols for numbers 0-20). For example, to represent the number seven, the Mayans would draw one bar and two dots just above the bar (the two dots would be side by side). To represent the number nineteen, the Mayans would draw three bars (one on top of the other) and four dots above the third bar (the four dots all lined up horizontally). The following is an example of what the numbers would look like when represented by dots and bars:



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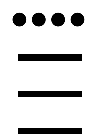

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Once the number twenty was reached, it would be represented by a single dot. Since the Mayas "used a systematic positional system, with twenty as the base of counting," they were able to avoid the confusion between, 1, 20, 400, 8000, and 160,000 (Lanczos, p. 17). They established several "floors" or "order" (Ifrah, p. 309). The numbers one through nineteen were placed on the "first floor," multiples of twenty were placed on the "second floor," multiples of 400 were placed on the "third floor," multiples of 8,000 were placed on the "fourth floor," and multiples of 160,000 would be placed on the "fifth floor" (Ifrah, p. 309). The same pattern would follow if greater numbers were involved. For example, to represent 21, the Mayans would place a dot for one on the first floor and a dot for 20 on the second floor, such that the two dots were aligned vertically. A bar and two dots in the "first floor" and three dots in the "second floor" would represent the number 77, equivalent to 3x20+17. To express the number 855, which is equivalent to 2x400+2x20+15x1, the Mayans would draw three bars in the "first floor," two dots in the "second floor," and 2 dots in the "third floor." For example:

However, what happened when the Mayans wanted to represent numbers such as 60, 500, or 8100? 60 is equivalent to 3x20, which contains no any units of the first floor/order. 500 is equivalent to 1x400+5x20, which also does not contain any units of the first floor/order. Moreover, 8100 is equivalent to 1x8000+5x20, which does not contain any units from the first floor nor the third floor. Thus, the ancient Maya invented a zero so that they could indicate that no units of a certain floor or order were being used to express a specific number. The Mayas discovered the zero in A.D. 500 (Salyers, p. 45). They used zero solely as a placeholder since they did not find any operational use for it (Lanczos, p. 17). Although it is claimed and widely accepted that the Hindus discovered the zero, Salyers argues that the Maya "discovered it independently of the Hindus" (Salyers, p. 45). Morely even claims that the Maya were the first ones to develop "man's first positional arithmetical system, one involving the concept of zero; this is among the most brilliant intellectual achievements of all time" (Morley, p. 454). The Mayan zero is represented by a figure that looks almost like an eye but it is sometimes represented as a "snail-shell" or "sea-shell" (Ifrah, p. 309). With the discovery of the zero, the Mayas could express numbers that were not composed of one of the powers of base 20. For example, 3 dots in the second floor, and a zero in the first floor would represent 60, which is equivalent to 3x20+0x1 in base 20. Five hundred, which is equivalent to 1x400+5x20+0x1, in base 20, would be represented with one dot in the third floor, one bar in the second floor, and a zero in the first floor. Also, 8100, which is 1x8000+0x400+5x20+0x1, would be represented with one dot in the fourth floor, a zero in the third floor, one bar in the second floor, and a zero in the first floor. (In the diagram below, a Θ represents zero, although the Maya used an "eye-shaped" figure or shell).

4th floor/order			●
3rd floor/order		●	Θ
2nd floor/order	●●●	▬	▬
1st floor/order	Θ	Θ	Θ
	60	500	8100

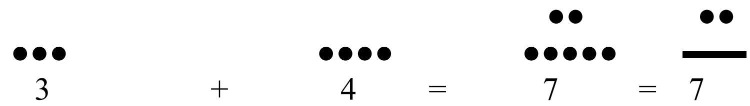
There has been a lot of debate among scholars as to whether the Mayas ever used their numerical system for everyday use and arithmetical purposes. According to Ifrah, the Mayas did not invent numbers for every day use such as trade. Lounsbury agrees that, unlike other civilizations, there is no evidence of the Mayan numerical notation ever being used in neither trade nor tribute (p. 764). He does mention, however, that there are cases in which the Maya enumerated objects for offerings" such as "nodules of copal, rolls of rubber, and cacao beans" (Lounsbury, p. 764). On the other hand, there are scholars who argue that the Mayas could add and subtract large numbers and their three symbols enabled them to perform these operations (Seidenberg, p. 380 & Lambert, Ownbey-McLaughlin, McLaughlin, p. 249).

In order to add, the Mayas combined the dots and bars as long as the quantities were of the same order. The Mayas only needed to remember three rules when adding. The first rule was that "a dot represents 1 unit;" second, "an accumulation of 5 dots is transcribed as a bar;" and third, "an accumulation of 4 bars in one [floor/order] becomes a dot in the next higher [floor/order]" (Lambert, Ownbey-McLaughlin, McLaughlin, p. 249). When four bars are exchanged for a dot in the next higher "floor or order" it is equivalent to carrying in our number system (Lambert, Ownbey-McLaughlin, McLaughlin, p. 249). However, with a vigesimal system, the Mayans would carry when they reached 20 instead of 10. The simplest arithmetic was adding two numbers of the first "floor/order." For example, to add seven and six, the following was done:

●●		●		●●●
▬	+	▬		▬
				▬
7	+	6	=	13

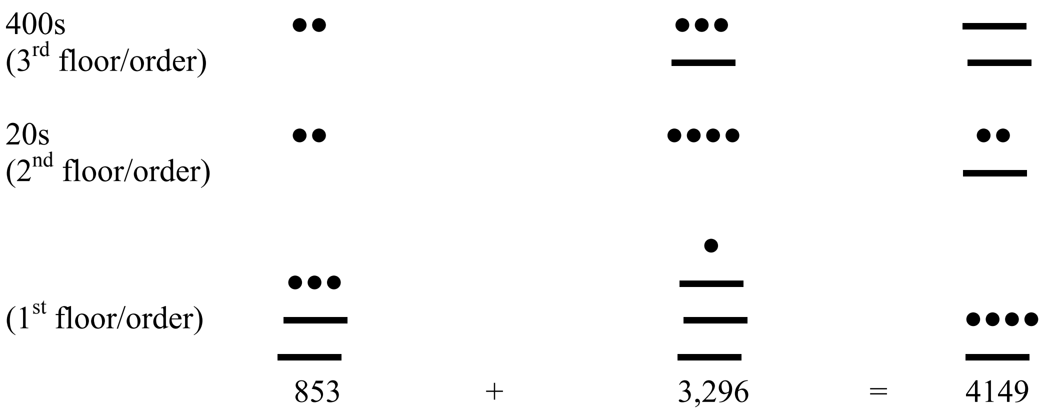
In this problem, all the bars were placed together, one on top of the other, and the three dots were placed together, immediately above the second bar.

We can see an application of the second rule when we add three and four:



Once the Maya placed all the dots together, they would see an accumulation of five dotes. Thus, a bar replaced the five dots and the two remaining dots were placed above the bar.

Adding became a bit more complicated when it came to adding larger numbers. The Mayans had to carry to the next higher floor/order once they reached 20 and they could only add numbers of the same order. For example, to add 853 and 3,296, the operation would be carried out in the following manner: First of all, 853=2x400+2x20+13x1, thus there would be 2 dots in the third "floor," 2 dots in the second "floor," and 2 bars and 3 dots in the first "floor." The number 3,296 is equivalent to 8 x 400 + 4 x 20+ 16 x 1, thus it would be represented by 1 bar and 3 dots in the third "floor," 4 dots in the second "floor," and 3 bars and 1 dot in the first "floor." The following diagram illustrates how 853 and 3,296 would be written in order to carry out the addition:



The Mayans would start adding from bottom to top. They would add the two quantities in the 1s place. When all the bars and dots are put together, there are 5 bars and 4 dots. By the rules, a total of four bars in the 1s place becomes a dot in the 20s place. Thus, a dot is carried over to the 20s place and one bar and 4 dots, which are equivalent to the number 9, remain in the 1s place. Now they would add the two quantities in the 20s place: 2 dots+4 dots=1 bar & 1 dot. Since a dot was carried over from the 1s place, it is added to the final quantity. As a result, we get 1 bar and 2 dots in the 20s place. In the third position (or 400s place) we have 2 dots + 1 bar & 3 dots which is equivalent to 5 dots + 1 bar. However, since 5 dots are equivalent to 1 bar, the final result is 1 bar + 1 bar = 2 bars. When the final quantities in the 400s, 20s, and 1s place are added together, the total is 4149, which is indeed the answer to 853 + 3,296. Thus, despite the fact that some scholars argue that Mayan numerical notation could not be used for adding, these examples demonstrate the contrary (Lambert, Ownbey-McLaughlin, McLaughlin, p. 249).

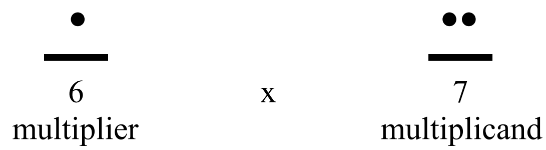
In the Mayan numerical system, subtraction could also be performed. In order to subtract, the Mayans would take the "difference of dots and bars in equal" floors/orders and borrow "when necessary from the next higher" floor/order (Lambert, Ownbey-McLaughlin, D. McLaughlin, p. 250). As opposed to the third rule under addition, the Maya would transfer 1 dot "as 4 bars into the next lower" floor/order when it was necessary to borrow. The following example from American Scientists illustrates how the Maya subtracted numbers (Lambert, Ownbey-McLaughlin, D. McLaughlin, p. 249):

The Mayans would start from the bottom just like with addition. In the 1s place we have 2 bars & 3 dots minus 1 bar & 4 dots. Since we cannot subtract 4 dots from 3 bars, we convert one bar into 5 dots, yielding the following: 1 bar & 8 dots minus 1 bar & 4 dots. The bars would "cancel each other out" and we are left with 8 dots minus 4 dots, which is equivalent to 4 dots. In the 20's place we have 2 bars & 3 dots minus 3 bars & 1 dot. Since 2 bars & 3 dots is less than 3 bars & 1 dot, we must borrow a dot from the 400s place as stated in the rules for subtraction. The dot gives us four additional bars in the 20s place and only four dots in the 400s place. Thus, the "new" quantity in the 20s place is 6 bars & 3 dots-3 bars& 1 dot, which yields 3 bars & 2 dots. Lastly, in the 400's place we have 4 dots minus 1 dot equals 3 dots. Thus, this shows that the Mayas were able to use the vigesimal system and their symbols to perform arithmetical operations such as subtraction. Their symbols facilitated the process of adding and subtracting because they could actually "see" the two quantities that were either being added or subtracted. When they were adding they could put all the symbols together and convert the total to bars and dots according to the rules for addition. When it came to subtraction, they could make the appropriate conversions as shown in the example above and cancel out the symbols.

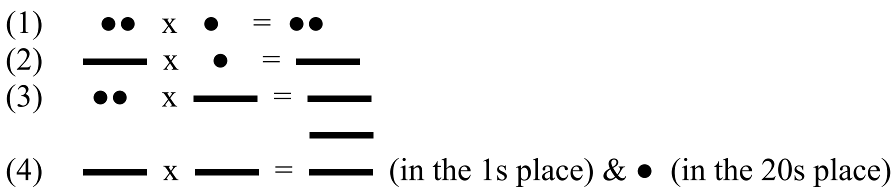
Scholars are still uncertain as to whether or not the Mayans used their notation to multiply and divide numbers. The Maya did have words for "'multiply,' 'multiplication,' 'division,' 'divide,'" but despite the knowledge of the words there is no assurance that the Maya actually used these operations (Lambert, Ownbey-McLaughlin, McLaughlin, p. 250). Many scholars, such as Thompson, claim that the Maya neither multiplied nor divided. But others, such as Satterthwaite, claim that just because there is no evidence that demonstrates that they did multiply or divide, does not mean that they did not carry out these operations with their notation (Lambert, O-McLaughlin, and McLaughlin, p. 249). The only piece of evidence that could give some kind of answer is the Dresden Codex, which is a document that was found in the Yucatan and has revealed information regarding Mayan math and scientific observations. Yet, there is no evidence in it that suggests that the Maya multiplied and/or divided using their numerical system. According to Lambert, Ownbey-McLaughlin, and McLaughlin, "it is doubtful that multiplication was used by the ancient Maya and almost certain that division was not used" (p. 255). It is possible that the Maya multiplied through successive addition and divided through successive subtraction (Lambert, Ownbey-McLaughlin, McLaughlin, p. 250). According to Seidenberg, "a Maya Priest could have multiplied 23,457 by 432, by repeated additions of 23,457" (p. 380). Thus, there is still not enough evidence that would give a definite answer as to whether or not the Mayans actually used their notation to multiply or divide.

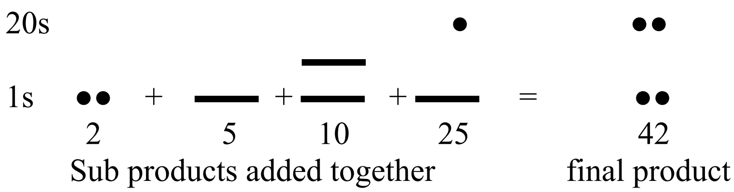
Even though there is no evidence that demonstrates that the Mayas used their notation for multiplication, Lambert, Ownbey-McLaughlin, and D. McLaughlin, suggest that it is possible to multiply using the Maya notation. With the knowledge of Arabic multiplication, scholars were able to come up with three rules to perform multiplication using the Maya notation. The first rule states that if 1x1=1, then dot x dot=dot; secondly, if 1x5=5, then dot x bar=bar; thirdly, if 5x5=25, then bar x bar=bar + dot (where the dot is in the next higher order) (Lambert, Ownbey, McLaughlin, p. 252). The following example illustrates the steps the Mayans could have taken to multiply two numbers in the "first order":

In Mayan notation 6 x 7 is expressed as:



The steps to find the product are as follows:

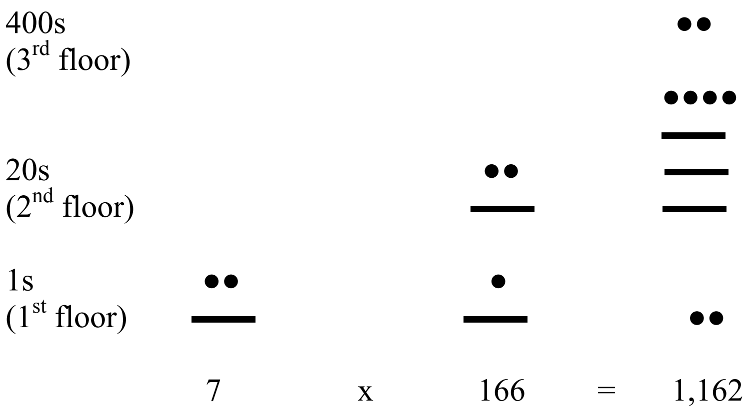




By following the rules for multiplication, we find that 2 dots in the multiplicand (i.e. 7) times 1 dot in the multiplier (i.e. 6) yields 2 dots; 1 bar in the multiplicand times 1 dot in the multiplier yields 1 bar; 2 dots in the multiplicand times 1 bar in the multiplier yields 2 bars; and, 1 bar in the multiplicand times 1 bar in the multiplier gives 1 bar in the 1s place plus 1 dot in the 20s place (the 2^{nd} higher order). Lastly, all the sub products are added together and the final result is 2 dots in the 1s place and 2 dots in the 20s place, which is equivalent to 42 (method to multiply learned using Lambert, Ownbey-McLaughlin, McLaughlin, p. 252).

Lambert, Ownbey, and McLaughlin also came up with method of multiplying larger numbers using the Mayan notation. Once again, they came up with three rules to carry out this operation. First of all, they defined m and n to be the floor/order number: "n and m are 1 for the units [floor/order], 2 for the 20s, 3 for the 400s, " and so on (Lambert, Ownbey, McLaughlin, p. 252). The first rule is that the product of a dot in any floor n with a dot in any floor m is going to be a dot in the floor right "below the one that corresponds to the sum of n and m" (p. 252). In other words, the product will be placed in the floor n+m-1. The second rule states that the product of a bar and a dot is a bar "that goes into the (n+m-1)" floor. The third rule states that the product of a bar and a bar is "a dot in the (n+m) floor and a bar in the (n+m-1) floor" (p.

252). For example, if the Mayans would have wanted to multiply 7 and 126, they would have followed the following procedures:



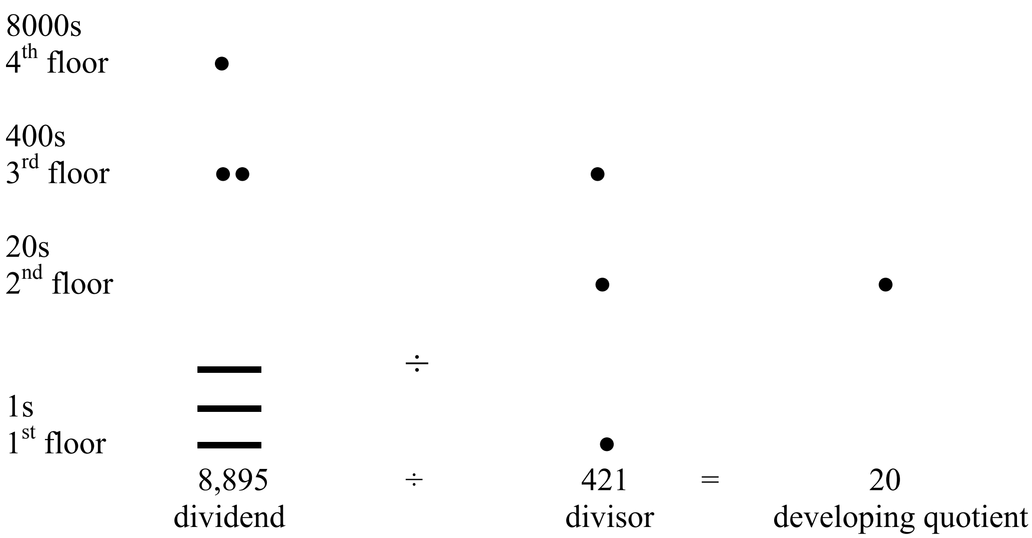
(1) Multiply the first floor numbers together (i.e. 7 and 6). In this case n=1 and m=1:
2 dots (in representation of 7) x 1 dot (in representation of 6)=2 dots in floor (1+1-1)=1
2 dots (in 7) x 1 bar (in 6)=2 bars in floor (1+1-1)=1
1 bar (in 7) x 1 dot (in 6)=1 bar in floor (1+1-1)=1
1 bar (in 7) x 1 bar (in 6)=1 dot in floor (1+1)=2 and 1 bar in floor (1+1-1)=1

(2) Next, they would multiply the first floor number, 7, by the second floor number 8, where 8 is actually 6x20. In this case n=1 and m=2.
2 dots (in 7) x 3 dots (in 8)=6 dots=1 bar and 1 dot in floor (1+2-1)=2
2 dots (in 7) x 1 bar (in 8)=2 bars in floor (1+2-1)=2
1 bar (in 7) x 3 dots (in 8)=3 bars in floor (1+2-1)=2
1 bar (in 7) x 1 bar (in 8)=1 dot in floor (1+2)=3 and 1 bar in floor (1+2-1)=2

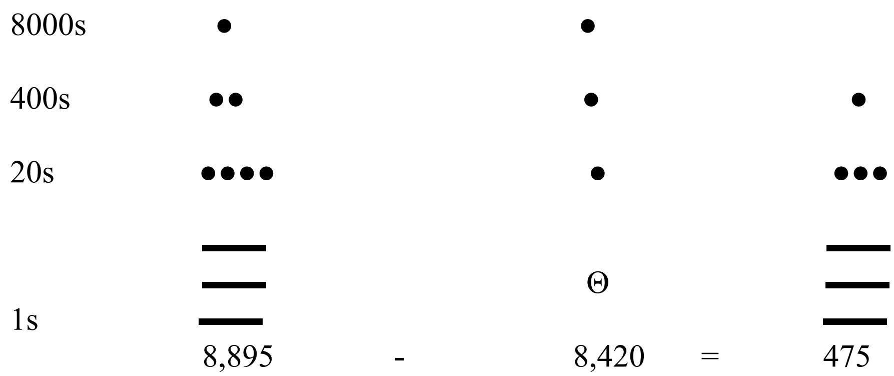
All of the results given above would be added by following the rules for addition mentioned earlier and the final result would be 2 dots in the 3rd floor (400s place), 3 dots and 3 bars in the 2nd floor (20s place), and 2 dots in the 1st floor (1s place), which gives a total of 1,162 or 2.18.2 in Mayan notation. Therefore, the method just described, demonstrates that it was possible for the Mayans to use their notation to multiply numbers together. It may be that the Mayas did multiply numbers together using their notation and the evidence may have been lost or destroyed. Perhaps they were unable to develop a method for multiplying numbers in the time period in which they lived, but there is reason to believe that they had the capability to discover a method similar to the one described above (Lambert, Ownbey-McLaughlin, McLaughlin, p. 252).

Whether or not the Mayas used their notation to divide numbers is still questionable. It seems highly probable that the wise men of the Maya civilization died or were killed before they could discover a method to divide numbers using their numerical system and notation. But once again, Lambert, Ownbey-McLaughlin, and D. McLaughlin, claim that the Mayan numerical notation was suited for dividing numbers. They came up with a procedure for dividing numbers that could be similar to the method used by the ancient Maya if they did in fact divide or, perhaps, the Maya could have developed if they had lived longer. Lambert, Ownbey-McLaughlin, and D. McLaughlin, demonstrate in five steps how division can be done with the problem 8,895÷ 421. The following problem is the one they used to show how to divide.

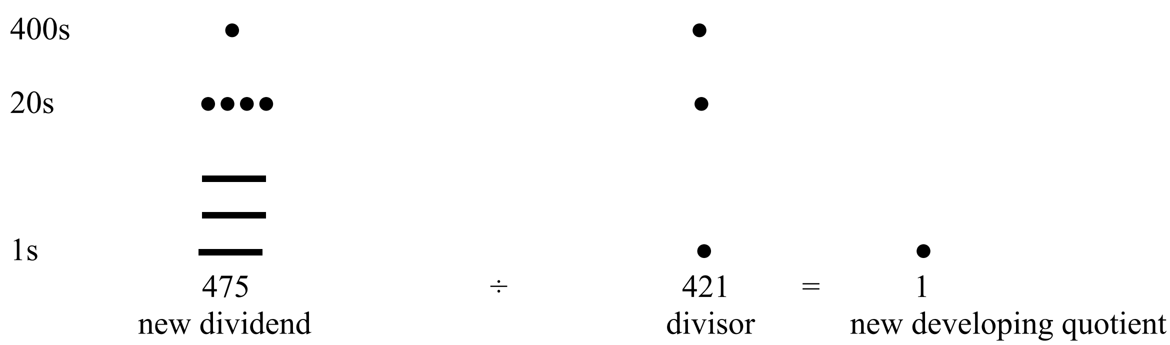
In the first step they express 8,895÷421 in Mayan notation.



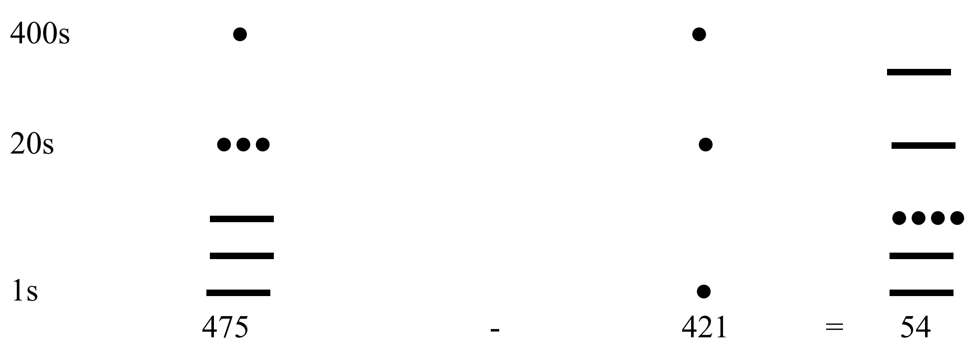
Since there is a dot in the 400s place of the divisor and a dot in the 8000s place of the dividend, using the knowledge of carrying out multiplication in the Mayan notation, we know that a dot in the 20s place times a dot in the 400s place gives a dot in the 8000s place (first rule under multiplication). Thus, a dot is placed in the 20s place of "the developing quotient" (p. 253). In the second step, the dot (representing 20) in the developing quotient is multiplied by the divisor, which yields 1 dot in the 20s place, 1 dot in the 400s place, and 1 dot in the 8000s place. This is equivalent to 8,420. Now 8,420 is subtracted from the dividend 8,895 as follows:



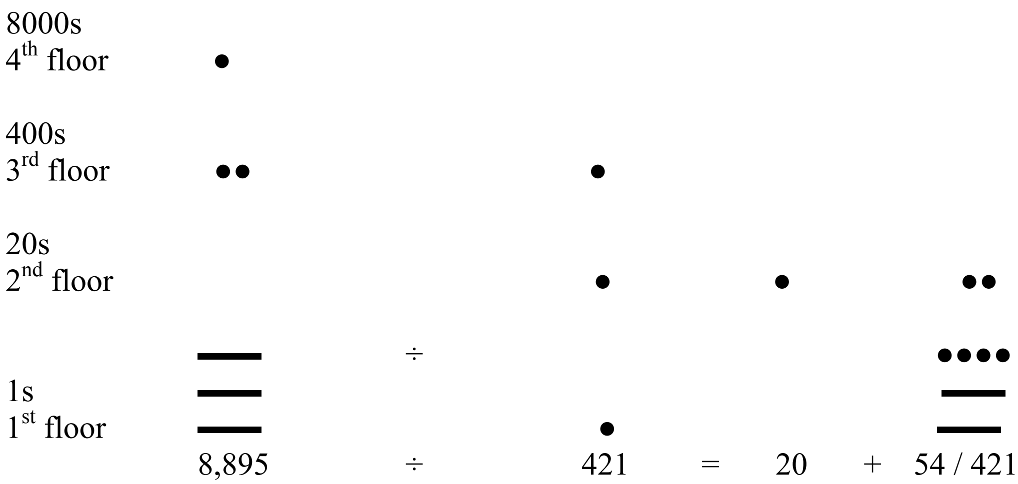
Thus, 475 is the remainder when 8,420 is divided by 421. Since 475 is greater than 421, 421 can still go into 475. Now 475 becomes the new dividend. 475 divided by 421 is illustrated as follows:



Once again, to divide 475 by 421, we think back to the rules of multiplication. Since a dot in the 1s place times a dot in 400s place yields a dot in the 400s place, a dot is placed in the 1s place of the new developing quotient. The remainder is calculated as in the second step, which gives a remainder of 54. Since 54 is less than the divisor, no more division operations can take place, so 54 is going to be the "remainder for the entire division operation" (p. 253).



Finally, the answer to 8,895 divided by 421 is going to be the sum of the developing quotients in the first and third steps and the remainder in the fourth step. The remainder is added with parenthesis to show that it is a remainder (p. 253). The illustration of the final result is as follows:



Through this example, it can be seen that the Maya numerical notation could be used to carry out division, although there is still no evidence to suggest that division was in fact part of the Mayas arithmetical operations. Many scholars argue that the Mayas did not use their numerical system to divide. Nevertheless, since their numerical system is appropriate for division operations, "it seems inevitable that Maya mathematicians would eventually have discovered these operations" (Lambert, Ownbey, McLaughlin, p. 255).

The ancient Maya used a purely vigesimal system when it came to counting, adding, and subtracting, Howe ever, they modified the vigesimal system in order to count time. In the pure vigesimal system, the place values are 1s, 20s, 400s, 8000s, 160,000s e.t.c.. However, the Mayans altered the system, using "a base of 18 rather than 20 for the third digit" (Lambert, Ownbey, McLaughlin, p. 249). Thus, the place values became 1s, 20s, 360s (instead of 400), 7200 (instead of 8000), 144,000s (instead of 160,000), e.t.c. (Salyers, p. 44). Instead of the third order being equivalent to 20 times the value of the second order (20x20=400), the value of the third order was equivalent to 18 times the value of the second order (18x20=360). The modification was only made in the third order because the fourth order was 20 times the value of the third order (20x360=7200), and the fifth order was 20 times the value of the fourth order (20x7200=144,000). To express the number 77, for example, they would still write it as

3x20+17x1, but to express the number 855, they would write it as 2x360+6x20+15x1 instead of 2x400+2x20+15x1. To express 13,110, using the modified vigesimal system, the Maya would write it as 1x7200+16x360+7x20+10x1, instead of 1x8000+12x400+15x20+10x1 using a purely vigesimal system. Morely claims "the break in the third order of units…360 instead of 400,….was used only in counting time" (Morely, p. 276).

According to Lambert, Ownbey, and McLaughlin, arithmetic operations can also be performed using the modified vigesimal system. In order to convert from pure vigesimal to modified vigesimal, also known as calendric vigesimal, two dots or two bars need to be added to "the next lower register [floor/order] for every dot or bar, respectively, in the third and higher registers [floors/orders]" (p. 254). However, there is no need to make any changes in neither the first nor the second floor. If these changes were not made to the pure vigesimal system, a vigesimal number "if interpreted calendrically, would be too small" (p. 254). Addition and subtraction using calendrical notation can be done the same way as with pure vigesimal notation. The dots and bars are combined together, keeping in mind that there can only be a "maximum of 18 units in the second" floor. For example, instead of 4 bars in the second floor becoming 1 dot in the third floor, once there are 3 bars and 3 dots in the second floor, they become 1 dot in the third floor. Unlike with the pure vigesimal system, there is no known method for multiplying and dividing using calendrical notation. Ifrah argues that the Mayas numerical system was suitable as a recording device but it had "no use for arithmetical operations" (Ifrah, p. 308). The Maya calendar is the only evidence where we can see that the Maya numerical notation was used. This suggests that the Mayans invented a numerical system for the purpose of "astronomical observation" and for counting time (Ifrah, p. 311).

Since the Maya were very religious, they also expressed numbers using the heads of the thirteen gods of the upper world. The primary numbers were zero through twelve. A distinct head that represented one of the thirteen gods expressed each number. The head for ten, for example, was the god of death. Each head also had distinct characteristics. For example, the head for six had a "hatchet eye," whereas the head for ten was known for its "fleshless lower jaw" and "truncated nose" (Morely, p. 103). The numbers thirteen through nineteen were represented by the head used for the primary numbers and the fleshless lower jaw of the head for ten. To represent sixteen, for example, the head would be that of six with the fleshless jaw of ten added to it (Salyers, p. 46).

The ancient Maya were not only great mathematicians but also brilliant astronomers. They were intelligent enough to modify the vigesimal system so that they could calculate time with more precision. One of their greatest achievements in astronomy is the invention of "one of the best calendars the world has ever seen" (Ifrah, p. 311). The Maya calendar was made up of 'three years.' One was known as the *Tzolkin* or "religious year," the second was the *Haab* or solar calendar year, and the third was the 360 "official year" or "Long Count" (Ifrah, p. 315).

The religious year calendar consisted of 260 days. The calendar for the religious year was called the "ritual calendar" and it was used for religious purposes. There were "twenty cycles of thirteen days" and it had "twenty named dates" (Ifrah, p. 312). Each day represented a deity, sacred animal, or sacred object (Ifrah, p. 312). The names of the days were: *Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etznab, Cauac,* and *Ahau*. Each day was assigned a number from one to thirteen. The first day was called Imix. However, since they had twenty named days, once the thirteen days had passed, the numbering would resume from the beginning. For example, the fourteenth day (Ix) would not be number 14, instead it would be number one, the fifteenth day (Men) would be number two, and so on. Thus, the twentieth day (Ahau) would actually be day number seven. After thirteen cycles, the numbering "came back to where it started, with day one counting once again as one" (Ifrah, p. 312). Thus, after 260 days, 1 Imix, would once again be 1 Imix (see table attached at the end for an example of the progression of the religious year).

In addition to the religious year, the Mayas had a solar year, known as the *Haab*. The solar year calendar consisted of "365 days divided into eighteen *uinal* (twenty-day periods)" (Ifrah, p. 313). A *uinal* was what we would call a month. Thus the Mayan months were made up of twenty days instead of thirty or thirty-one like in our calendar. Since the 365 days were divided into 18 *uinal*, there was a period of five days that remained at the end of the eighteenth *uinal.* The names of the 18 *uinal* or months were: *Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku* (Ifrah, p. 313). The remaining five days were categorized as *Uayeb*. According to Ifrah, *Uayeb* meant "The one that has no name" (p. 314). The Maya believed that these days were "ghost days" and anyone born in these days would "have bad luck and remain poor and miserable" throughout their entire life (Ifrah, p. 314). The days of the month were numbered from zero to nineteen. The zero day was actually the first day of the month and the nineteenth day was actually the twentieth day of the month. To keep track of the days, the Maya described each day by the number and the month. For example, each day in the month *Pop* would be called *0 Pop, 1 Pop, 2 Pop,…,19 Pop*. In the five day period of *Uayeb*, the days would be *0 Uayeb, 1 Uayeb, 3 Uayeb, 4 Uayeb,* and *5 Uayeb* (Ciudad, p. 102). The hieroglyphs that represent each day resemble animals. Whereas the ritual calendar served a religious purpose, the solar calendar served agricultural purposes.

Not only did the Maya have the ritual year and the solar year, but also the Calendar Round. Since the Maya used the religious calendar together with the solar calendar, they expressed a specific date by its name and number in the religious calendar and its name and number in the solar calendar (Ifrah, p. 315). For example, Ifrah shows that 13 *Ahau* in the religious year is equivalent to *18 Cumku* in the solar year. Thus the Calendar Round is considered to be the amount of time it takes for a "given date in the solar calendar to match a given date in the religious or ritual calendar" (Ifrah, p. 315). The Mayans calculated that after 18,980 days, which is the least common multiple of 365 and 260, a given date in the religious calendar and in the solar calendar would once again be the same. This means that it would take 52 solar years and 73 religious years. This shows the vast knowledge that the Mayans had of mathematics and how they were able to make great use of it in their astronomical observations. Moreover, it shows that the Maya had some notion about infinity or perhaps of "boundless, unending stretch of time" (Ifrah, p. 298).

The last calendar that the ancient Maya had was called the "Long Count." Since the Maya wanted to count every single day since the beginning of the Mayan era, they developed the Long Count. For this reason, the Long Count starts at day zero. In the Long Count, a year lasted 360 days. The unit was the *kin* or day, 20 *kins* was equivalent to 1 *uinal* (20 days or 1 month)*,*18 *uinals* was equivalent to 1 *tun* (1 year or 360 days), 20 *tuns* was equivalent to 1 *katun* (20 years), 20 *katuns* was equivalent to 1 *baktun (*400 years), and 20 *baktuns* was equivalent to 1 *pictun* (8000 years) (Ifrah, p. 36). The ancient Maya knew the "names and numerical values" up to 18x20^9. Thus they were capable of computing time up to 23,040,000,000 days or more. The following sequence of numbers was found in the codices of the Maya: 9.16.4.10.8. This number represents 8 *kins* (days), 10 *uinals* (months), 4 *tuns* (years), 16 *katuns*, and 9 *baktuns* (Seidenberg, p. 381). This shows how the Maya would express the amount of time that had gone by since day zero.

The way the Mayans calculated the passage of time follows the modified vigesimal system presented earlier. Instead of continuing with the vigesimal pattern and having twenty *uinals* equivalent to 1 *tun,* the Maya established that eighteen *uinals* (months) was equivalent to 1 *tun* (or year). If the ancient Maya had made twenty *uinals* equivalent to 1 *tun*, thereby following a pure vigesimal system, then their year would have 400 days. However, since the solar year is 365 days, 360 days was a closer approximation to it than 400 days. For this reason, the Maya adjusted the vigesimal system in the third order to 18 (instead of 20) times the value of the second order. Moreover, the use of a vigesimal system enabled them to express large periods of time.

One of the most interesting aspects about the ancient Maya is that they were able to count time with great precision and come up with various calendars without the technology that modern day mathematicians and scientists use to carry out these calculations. The Maya were able to calculate that the solar year lasted 365.242 days and in actuality modern day astronomers have found that the solar year lasts 365.242198 days (Ifrah, p. 297). To come up with the length of the solar year, the Maya used a tool known as a *gnomon* (Ifrah, p. 298). It is a rigid stick that is placed at the center of a flat surface. Since the stick casts a shadow, as the day goes by the shadow changes. When the shadow is "at its shortest then the sun has reached its highest point above the horizon, and it is true non" (Ifrah, p. 298). The ancient Maya also used their buildings to make their astronomical observations. The Maya were able to measure the lunar cycle as well. According to Ifrah, the Mayans observed the moon for 2,329 days and found that throughout this time period, there were 81new moons. Thus, they calculated that the lunar cycle lasted 29.53086 (Ifrah, p. 297). Thus, by merely observing the sky and using the simplest tools, the Maya were able to make more accurate measurements of the solar and lunar cycles than the astronomers of the time and even later astronomers.

As we can see, the ancient Maya were not as "barbaric" as the Spanish colonizers and other European nations believed and scholars may have presumed before learning about their mathematical and scientific achievements. The ancient Maya civilization was very wise and intelligent. Unlike other number systems, the Mayan number system was developed for specific purposes. They used a pure vigesimal system for counting and arithmetic operations and a modified vigesimal system for counting time. Moreover, a vigesimal system facilitated the expression of very large numbers. Unlike the symbols used by other civilizations, their symbols for numbers, known as the "eye" or "shell," the dot, and the bar, were much easier to remember, to count with, and perform arithmetical operations. Although there is still not enough evidence to conclude that the Maya used their notation to multiply and divide numbers, their number system was capable of being used to perform these operations. If they did not already multiply or divide, they would have surely come up with a method to perform these operations. The Mayas used their mathematical knowledge for their astronomical observations as well. They used their modified vigesimal system to calculate the passage of time and come up with the most sophisticated calendar. Moreover, their scientific discoveries were made by mere observations of the sky and the simplest tools and were almost equivalent to the discoveries made by astronomers who had technology at their disposal. Thus, the ancient Maya were great mathematicians and scientists and, perhaps, if they had lived longer, they would have made more mathematical and scientific discoveries.

Figure 1. The Mayan symbols for numbers 0 through 19 (Ifrah, p. 309)

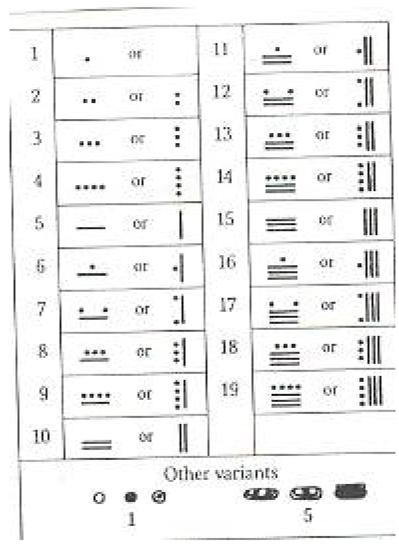


Figure 2. Numbers Represented by the Heads of the Gods of the Upper World (Ifrah, p. 317)

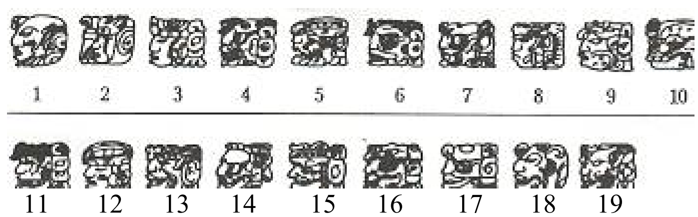
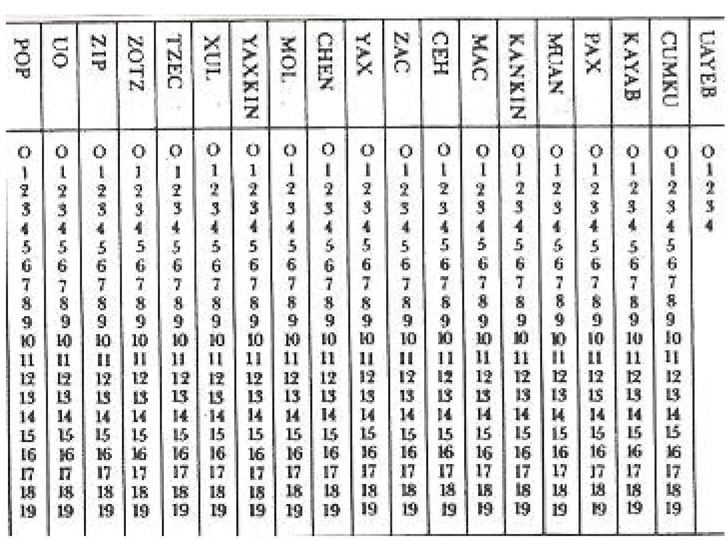


Figure 3. The Mayan 365-day Solar Year (Ifrah, p. 315)



POP	UO	ZIP	ZOTZ	TZEC	XUL	YAXKIN	MOL	CHEN	YAX	ZAC	CEH	MAC	KANKIN	MUAN	PAX	KAYAB	CUMKU	UAYEB
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	
13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	
17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	
18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	
19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	

Figure 4. The Mayan 260-day Religious Year (Ifrah, p. 313)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
IMIX	1	8	2	9	3	10	4	11	5	12	6	13	7
IK	2	9	3	10	4	11	5	12	6	13	7	1	8
AKBAL	3	10	4	11	5	12	6	13	7	1	8	2	9
KAN	4	11	5	12	6	13	7	1	8	2	9	3	10
CHICCHAN	5	12	6	13	7	1	8	2	9	3	10	4	11
CIMI	6	13	7	1	8	2	9	3	10	4	11	5	12
MANIK	7	1	8	2	9	3	10	4	11	5	12	6	13
LAMAT	8	2	9	3	10	4	11	5	12	6	13	7	1
MULUC	9	3	10	4	11	5	12	6	13	7	1	8	2
OC	10	4	11	5	12	6	13	7	1	8	2	9	3
CHUEN	11	5	12	6	13	7	1	8	2	9	3	10	4
EB	12	6	13	7	1	8	2	9	3	10	4	11	5
BEN	13	7	1	8	2	9	3	10	4	11	5	12	6
IX	1	8	2	9	3	10	4	11	5	12	6	13	7
MEN	2	9	3	10	4	11	5	12	6	13	7	1	8
CIB	3	10	4	11	5	12	6	13	7	1	8	2	9
CABAN	4	11	5	12	6	13	7	1	8	2	9	3	10
ETZNAB	5	12	6	13	7	1	8	2	9	3	10	4	11
CAUAC	6	13	7	1	8	2	9	3	10	4	11	5	12
AHAU	7	1	8	2	9	3	10	4	11	5	12	6	13

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