Applied Algebra Qualifying Exam: Part C

1:00pm-4:00pm (PDT). Thursday September 7th, 2023

- Write your name and student PID at the top right corner of each page of your submission.
- Show your work.
- This part of the exam will represent 20% of the total score.
- By participating in this exam you are agreeing to abide by the UCSD Policy on Academic Integrity. The instructors reserve the right to require a follow-up oral examination.
- This is a closed-book examination. No cell-phone or Internet aids.

Question 1. Let **H** be an *n*-dimensional Hilbert space and $A: \mathbf{H} \to \mathbf{H}$ a normal linear transformation.

(a) (5 points) Derive a formula for the trace of the degree d exterior power $A^{\wedge d}$ as a function of the eigenvalues of A.

(b) (5 points) Express the coefficients of the characteristic polynomial of A in terms of traces of exterior powers of A.

Question 2. Consider the monomial ideal I = ⟨x₂²,...,x_n²⟩ in C[x₁,...,x_n].
(a) (3 points.) Give a basis of the vector space C[x₁,...,x_n]/I.

(b) (4 points.) Compute the Hilbert function of the ideal I.

(c) (3 points.) Determine the affine variety V(I), and explain whether or not it is reducible.