Applied Algebra Qualifying Exam: Part C

1:00pm–4:00pm (PDT).
Thursday September 7th, 2023

• Write your name and student PID at the top right corner of each page of your submission.

• Show your work.

• This part of the exam will represent 20% of the total score.

• By participating in this exam you are agreeing to abide by the UCSD Policy on Academic Integrity. The instructors reserve the right to require a follow-up oral examination.

• This is a closed-book examination. No cell-phone or Internet aids.
Question 1. Let $H$ be an $n$-dimensional Hilbert space and $A : H \to H$ a normal linear transformation.

(a) (5 points) Derive a formula for the trace of the degree $d$ exterior power $A^{\wedge d}$ as a function of the eigenvalues of $A$. 
(b) (5 points) Express the coefficients of the characteristic polynomial of $A$ in terms of traces of exterior powers of $A$. 
Question 2. Consider the monomial ideal $I = \langle x_2^2, \ldots, x_n^2 \rangle$ in $\mathbb{C}[x_1, \ldots, x_n]$.

(a) (3 points.) Give a basis of the vector space $\mathbb{C}[x_1, \ldots, x_n]/I$. 
(b) (4 points.) Compute the Hilbert function of the ideal $I$. 
(c) (3 points.) Determine the affine variety \( V(I) \), and explain whether or not it is reducible.