Integration Strategy

General tips:

1. **Start simple!** Compare the integrand with derivatives of known functions (some scary looking things like $\frac{1}{\sqrt{1-x^2}}$ are really easy to integrate!) See if you can algebraically simplify the integrand. See if there is a simple substitution that will work. Don't launch straight into the hardest technique you know!

2. **It's okay to use multiple steps/techniques!** For example, it's totally okay to use two substitutions in a row, or to use integration by parts followed by substitution. Sometimes there are longer or shorter ways to solve a problem, but there is not always a "best" way, and a lot of it just boils down to what's easiest and most comfortable for you.

3. **Try again!** If something didn't work, try a different technique. Sometimes the only way to know if an approach will work is to give it a shot, so try different things, and don't be afraid of hitting dead ends.

Learning tips:

1. Learn each technique thoroughly, then focus on the strategy-building shown here.

2. Don't mistake your ability to do these problems "in-section" for doing them on a quiz/exam: for example, it's easy to know to use integration by parts when you're in the integration by parts section of the book. One good source of "unlabeled" practice integrals is the integral list on the website: https://mathweb.ucsd.edu/ gmckinley/10B_f22/integralSheet.pdf

Overview of integration techniques on the <u>next page</u>!

Overview of Techniques

Overview of the integration techniques we've learned and when they are useful.

1. **Direct integration:** You can immediately do the integral (or after some algebraic simplification/manipulation).
   Examples: $\int \sec^2 x\,dx, \quad \int \frac{\sqrt{y}+ye^y}{y}\,dy, \quad \int \sqrt{t}(t-1)\,dt$

2. **u-Substitution:**
   - (Typical) You see a function and its derivative in the integrand. The problem collapses dramatically.
     Examples: $\int e^{\sin x}\cos x\,dx, \quad \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx, \quad \int \frac{x}{x^2-1}\,dx$
     (Pro tip: that last integral could be done with partial fractions as well, but u-substitution is much simpler!)
   - (Typical) The integrand is very close to the derivative of a known function, but off by some constants.
     Examples: $\int \frac{1}{\sqrt{9-16x^2}}\,dx, \quad \int e^{5x+1}\,dx, \quad \int \frac{1}{4x^2+1}\,dx, \quad \int \sin(2x-7)\,dx$
   - (Less typical, but very effective!) By "shifting" the variable of integration slightly, (e.g., letting $u = x-1$ or $u = 2x+1$), you can algebraically simplify the integrand, and the problem becomes much easier.
     Examples: $\int x(1-x)^{99}\,dx, \quad \int x^2\sqrt{x-2}\,dx$
     (Pro tip: either of these examples could also be done with integration by parts, but u-substitution will be simpler!)

3. **Integration by Parts:** Split the integrand into u and $\frac{dv}{dx}$. Good in several cases:
   - A polynomial times a function.
     Examples: $\int x^2e^x\,dx, \quad \int 3x\cos x\,dx, \quad \int x^4\ln x\,dx$
   - A function whose derivative is simpler.
     Examples: $\int \ln x\,dx, \quad \int \tan^{-1} x\,dx$
   - The product of two functions whose derivatives loop around.
     Example: $\int e^x\cos x\,dx$

4. **Partial Fractions:** Used when you have a rational function. Remember that long-division might be needed.
   Examples: $\int \frac{x^4-3x+1}{x^3-x}\,dx, \quad \int \frac{3}{(x-2)^3(x^2+4)}\,dx$