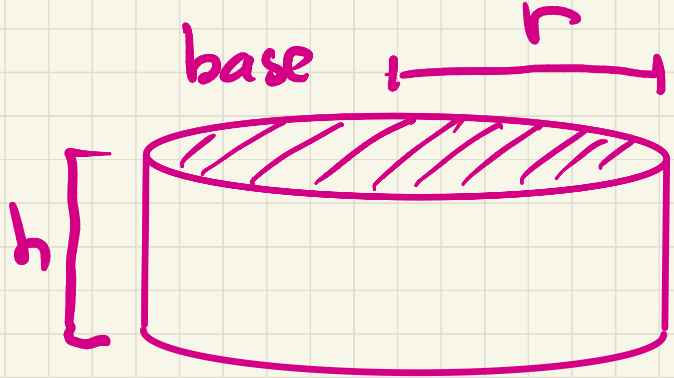
The "Saturno" table by Brazilian designer Fernando Jaeger is made of circular plywood disks.

How would you find the volume of this table?

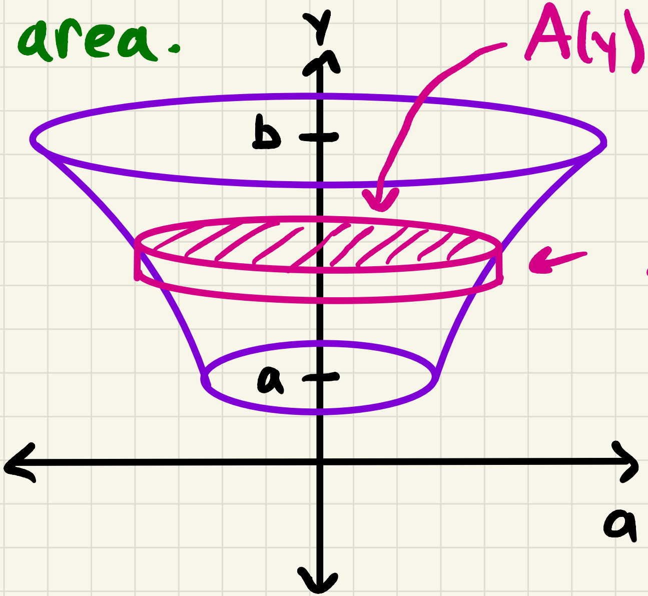
Find the volume of each disk, and add them all up (summation).

Volume of cylinder =
(area of base)(height)
$(\pi r^2)\ h$



Today: Ch. 2.2 Find volume by slicing

Idea: Find the volume of a solid by taking thin slices with almost constant cross-sectional area.



$A(y)$ = area of cross-section

Δy = thickness of slice

$a \le y \le b$

Area:

$$\lim_{n\to\infty} \sum_{i=1}^{n} A(y_i)\Delta y$$

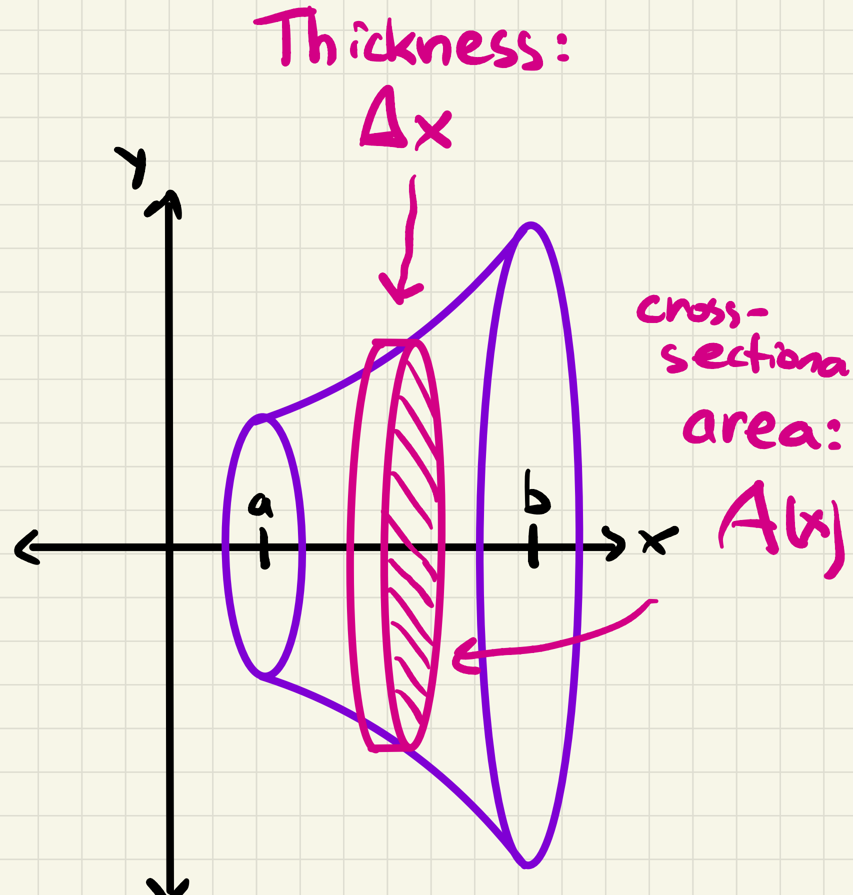
$$= \int_a^b \underbrace{A(y)}_{\text{area of slice}}\, dy$$

We can do the same thing with vertical instead of horizontal slices.

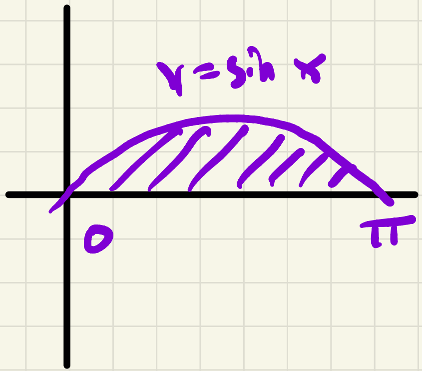
Area:

$$\lim_{n \to \infty} \sum_{i=1}^{n} A(x_i)\,\Delta x$$

$$\int_a^b A(x)\,dx$$



Example: Take the region between $y = \sin x$ and $y = 0$ from $x = 0$ to π, and rotate it around the x-axis to produce a solid of revolution.

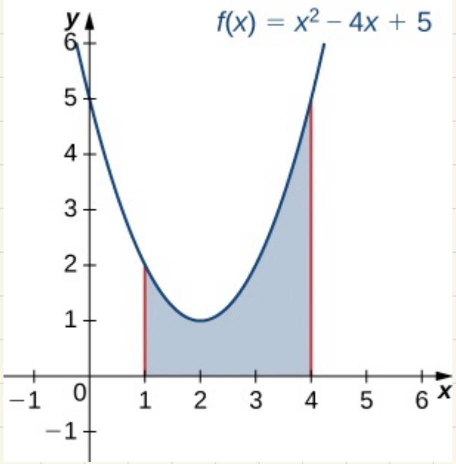


Cross-sectional area: $A(x) = \pi (\sin x)^2$

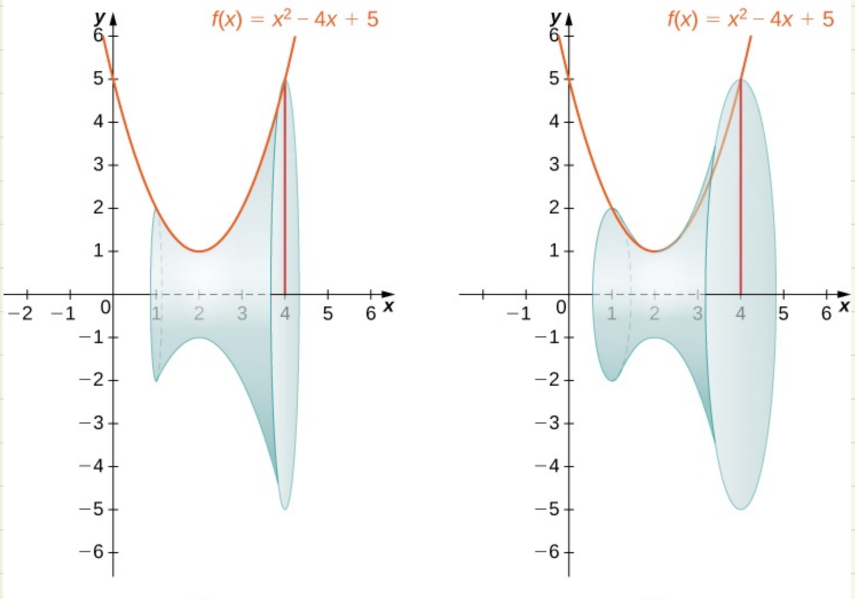
Volume: $\int_0^{\pi} \pi (\sin x)^2 \, dx$

Example 2.7

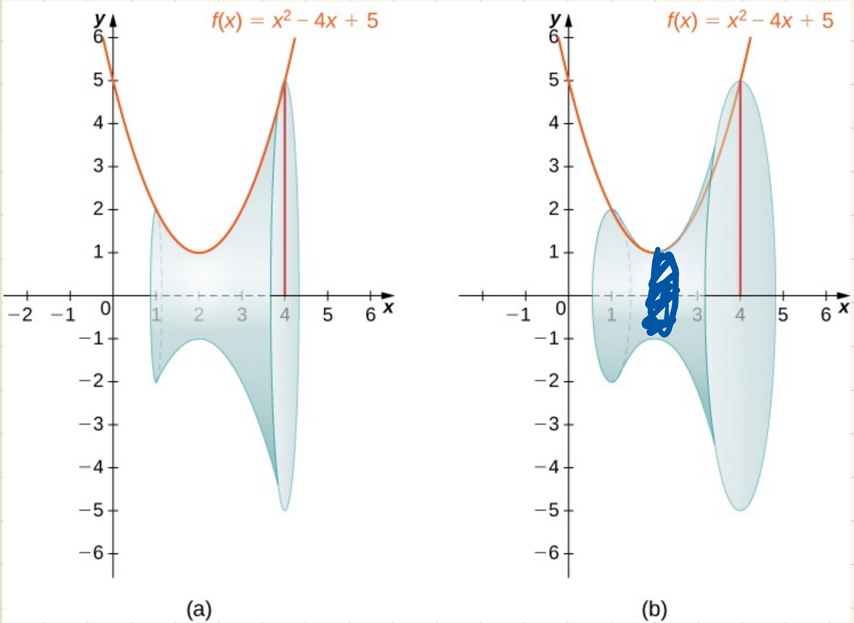
Consider the region between the x-axis, $y = x^2 - 4x + 5$, $x = 1$, and $x = 4$. Rotate this around the x-axis to produce a solid of revolution.



Use integration to find the volume!



(a) (b)



Slice = circle of radius $f(x) = x^2 - 4x + 5$

$$A(x) = \pi (x^2 - 4x + 5)^2$$

$$\int_1^4 \pi (x^2-4x+5)^2\, dx = \pi \int_1^4 (x^2 - 4x + 5)^2\, dx$$

$$= \pi \int_1^4 (x^4 - 8x^3 + 10x^2 + 16x^2 - 40x + 25)\, dx$$

$$= \pi \left(\frac{1}{5}x^5 - 2x^4 + \frac{10}{3}x^3 + \frac{16}{3}x^3 - 20x^2 + 25x \right)\Big|_{x=1}^{x=4}$$

RULE: Disk method for solids of revolution:

Around x-axis: Let $f(x) \geq 0$ continuous, for $a \leq x \leq b$.

Region R where $a \leq x \leq b$, $0 \leq y \leq f(x)$.

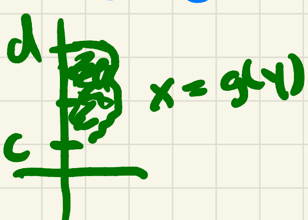
Volume of solid of revolution of R around x-axis:

$$V = \int_a^b \pi \, f(x)^2 \, dx$$

Around y-axis: Let $g(y) \geq 0$ continuous, for $c \leq y \leq d$.

Region Q where $c \leq y \leq d$, $0 \leq x \leq g(y)$.

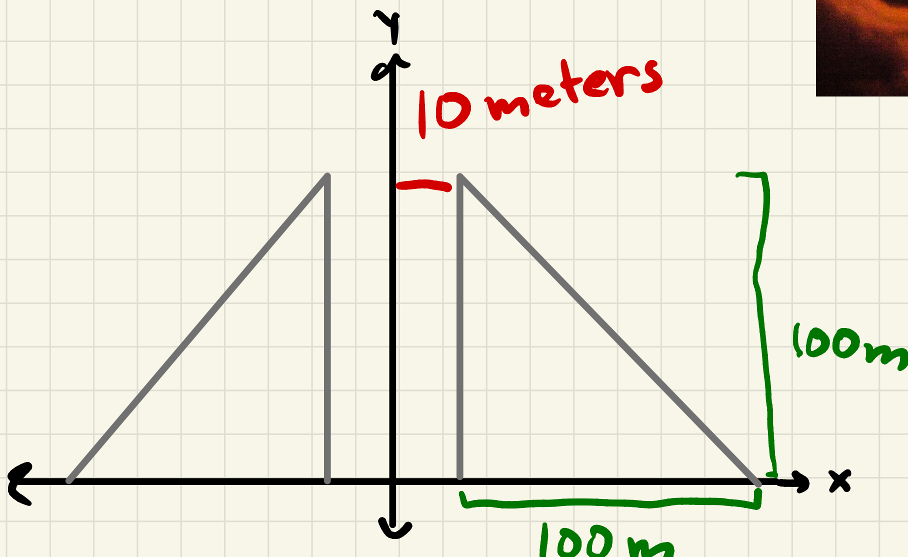
Volume of solid of revolution of Q around y-axis:



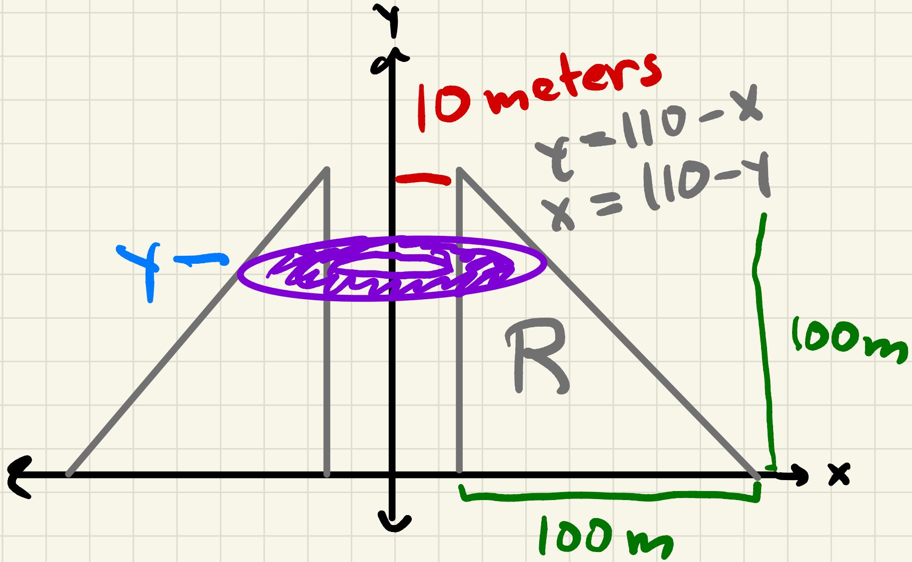
$$V = \int_c^d \pi \, g(y)^2 \, dy$$

Example: You want to find the volume of a cone-shaped volcano with a hole in the middle.

(Image: German Adventurer
Getty Images)



We consider the volcano as a solid of revolution about the y-axis.



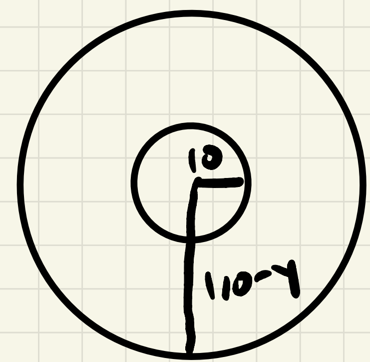
Describe the region R with inequalities:

$$0 \le y \le 100$$

$$10 \le x \le 110 - y$$

What shape is the horizontal slice of the volcano at height y?

Washer – circle of radius $110-y$ minus a circle of radius 10.



What is the area of the slice?

$$\pi (110-y)^2 - \pi (10)^2$$

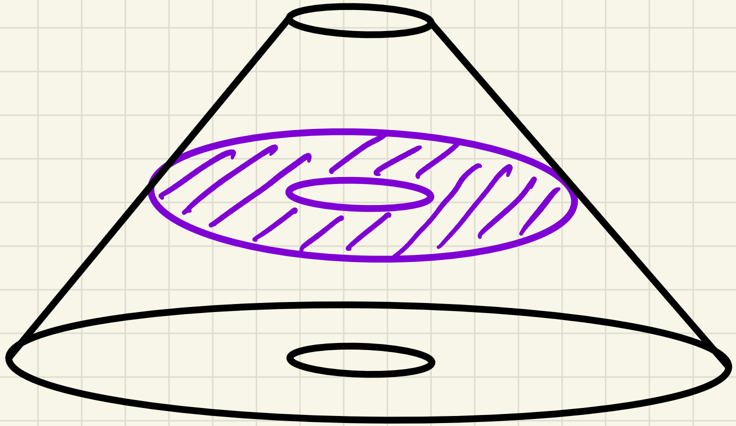


Image: Lowe's

$$\int_0^{100} A(y)\,dy$$

$$= \int_0^{100} \pi\left((110-y)^2 - 10^2\right) dy = \pi\int_0^{100}(y-110)^2\,dy - \pi\int_0^{100} 100\,dy$$

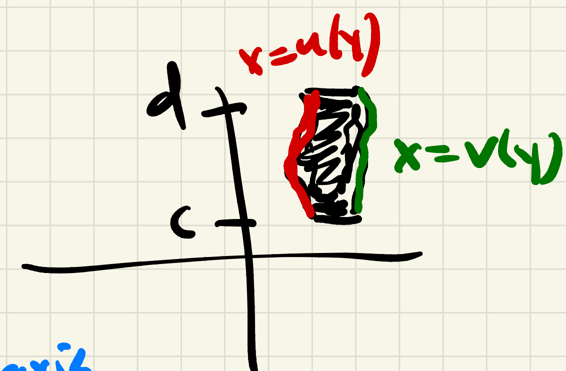
$u = y-110$
$du = dy$

$$= \pi\int_{-110}^{-10} u^2\,du - \pi\int_0^{100} 100\,dy = \pi\left[\tfrac{1}{3}u^3\right]_{-110}^{-10} - \pi\, 10{,}000$$

Rule: Washer method for solids of revolution about the y-axis:

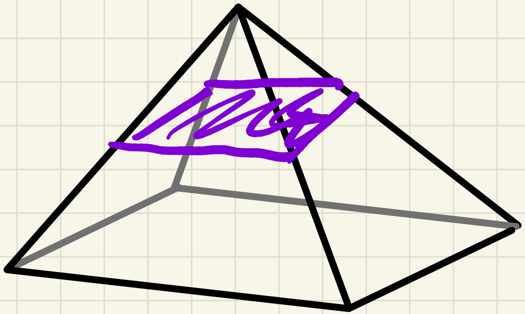
Suppose $0 \le u(y) \le v(y)$ continuous.
Q region where $c \le y \le d$, $u(y) \le x \le v(y)$.
The volume of the solid of revolution formed by rotating Q around the y-axis is

$$V = \int_c^d \pi\left(v(y)^2 - u(y)^2\right) dy$$

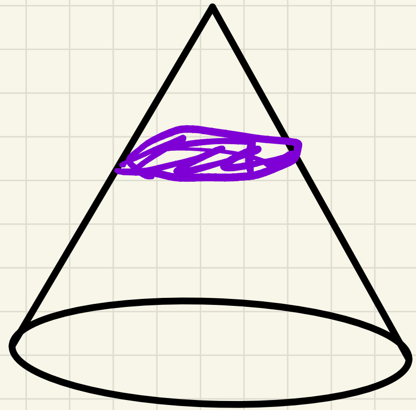


See book: Same statement but revolve around x instead of y-axis.

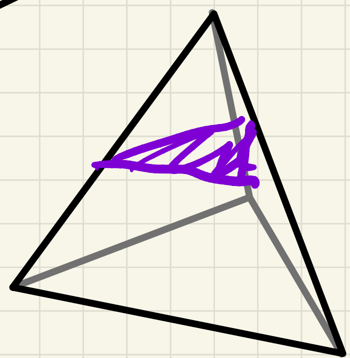
More applications of finding volume by slicing!



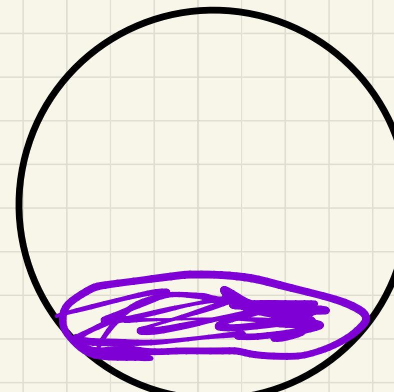
Pyramid



Cone

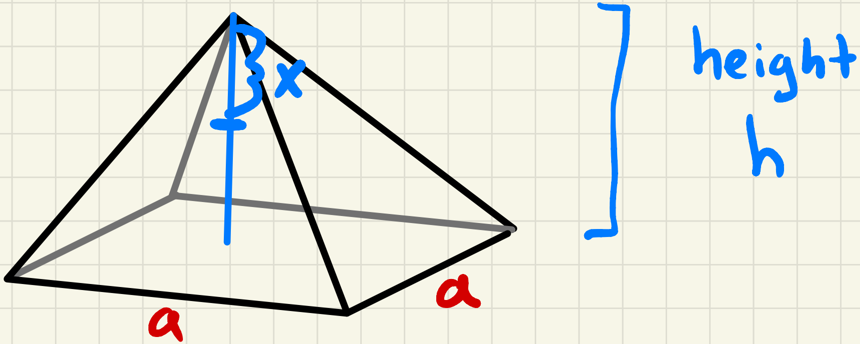


Tetrahedron

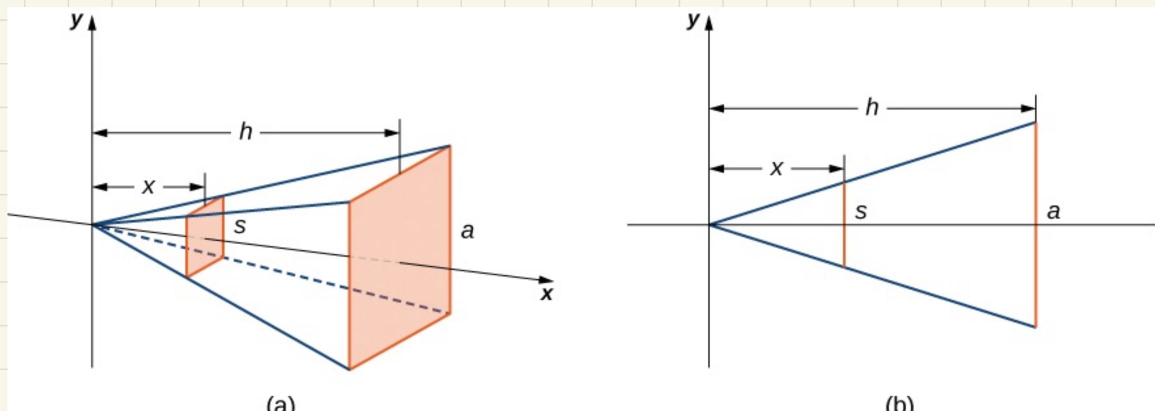


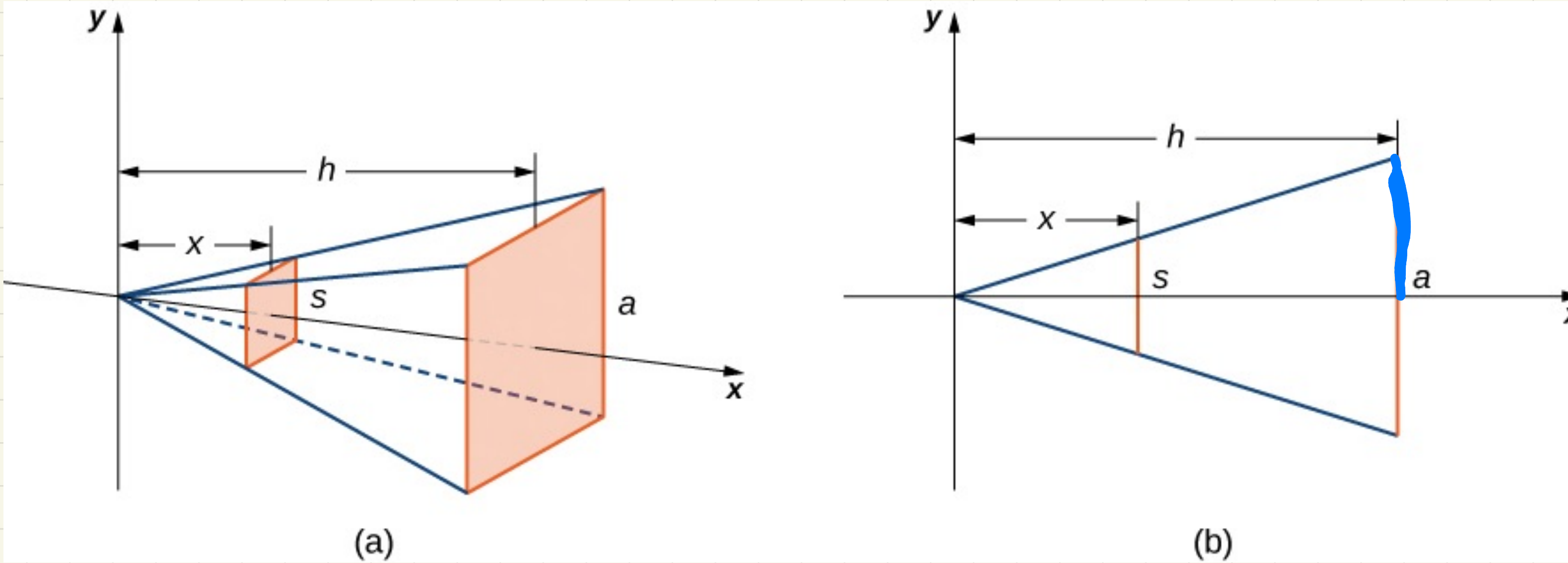
Sphere

Example 2.6: Volume of a pyramid



side length





s = side length of slice for x

Similar triangles: $\dfrac{s}{x} = \dfrac{a}{h}$

$s = \dfrac{a}{h}x$

$A(x) = \left(\dfrac{ax}{h}\right)^2$

Conclusion: x ranges from 0 to h

Cross-sectional area is $\left(\frac{ax}{h}\right)^2$

Volume is $\int_0^h \left(\frac{ax}{h}\right)^2 dx$

More Resources: See Class Schedule Page

Example videos from Dr. McKinley

- Disk method, revolution around x-axis
- Disk method, y-axis
- Washer method, x-axis
- Washer method, y-axis

Link to Geogebra Demo from earlier

Announcements:

① Quiz 2 next Friday October 28.

② Bonus point on next week's homework if you work together with two classmates.

③ Reminder: "Review" problems on homework <u>are</u> graded.

④ Today is the deadline to drop courses without a W on transcript.