Ex Can you buy a house? San Diego:

- $750,000 loan ← $850K – $100K (house) (down-payment)
- 30 years (360 months)
- 6% interest/year (actually .5%/month) ← .005 as a decimal

actually slightly higher! ↗

Divided into 360 monthly payments of $m each.

- Find m
- How much do you pay <u>total</u> over 30 years?

Take a guess – write it down!

Write $P = 750,000$ ("principal")

Amount owed initially $= P$

P grows to $(1.005)P$ in 1 mo

Amt owed after 1 month $= (1.005)P - m$

pay off this much at the end of the month

after 2 months $= (1.005)[\quad] - m$

$= (1.005)^2 P - (1.005)m - m$

after 3 months $= (1.005)[\quad] - m$

$= (1.005)^3 P - (1.005)^2 m - (1.005)m - m$

$\vdots$

after n months $= (1.005)^n P - (1.005)^{n-1} m - \cdots - (1.005)m - m$

$= (1.005)^n P - \sum_{k=0}^{n-1} (1.005)^k m$

partial sum S_{n-1} of geom series with $r = 1.005$, $a = m$

Recall

$$S_n = \frac{a(1-r^{n+1})}{1-r}$$

$$= (1.005)^n P - \frac{a(1-r^n)}{1-r}$$

$$= (1.005)^n P - \frac{m(1-(1.005)^n)}{1-1.005}$$

Amount owed = 0 after 30y (360 mo)

(It's a 30-year mortgage)

$$0 = (1.005)^{360} P - \frac{m(1-(1.005)^{360})}{1-1.005}$$

Solve for m:

$$m = \frac{1-1.005}{1-(1.005)^{360}} (1.005)^{360} P$$

calculator (remember: P = 750,000)

$$\approx \boxed{\$4,497}$$

← monthly payment

total # of payments

Total amount paid for mortgage $\approx \$4500/\text{mo} \cdot \boxed{360 \text{ mo}}$

$= \boxed{\$1,619,000}$

(more than double the principal!)