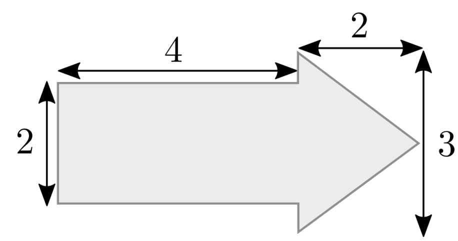
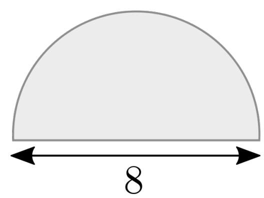
Math 10B Website Problems

W1. Find the area of the figure below.



W2. Find the area of the figure below.



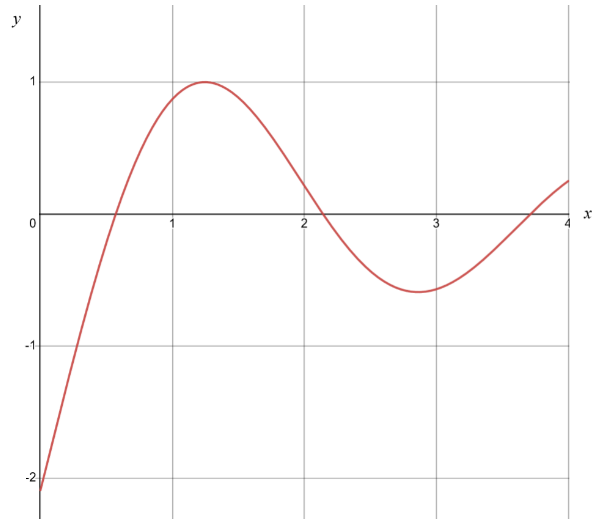
W3. Determine whether the hypotheses of the Mean Value Theorem hold for the function f on the given interval, and if they do, find all values of c in the interval satisfying the conclusion of the Mean Value Theorem.

$$f(x) = 3x^2 - 5x + 1 \quad \text{on } [2, 5]$$

W4. Evaluate the following sum: $\displaystyle\sum_{k=0}^{5} 2k$

W5. Rewrite the following expression as a polynomial without summation notation (i.e., expand the summation notation). $\displaystyle\sum_{i=3}^{5} i^2 x^{3i-1}$

W6. The graph of a function f is given below. Make a rough sketch of the antiderivative F of f that has $F(0) = 2$. Explain your work.



W7. Find the most general antiderivative of $f(t) = -10\cos(t) + 5\sin(t)$.

W8. Consider the function

$$f(x) = 10\cos(x) - 2\sin(x).$$

Let $F(x)$ be the antiderivative of $f(x)$ with $F(0) = 1$. Find $F(x)$.

W9. Find the particular antiderivative R that satisfies the following conditions:

$$\frac{dR}{dt} = \frac{70}{t^2}, \quad R(1) = 10.$$

W10. Find the function $h(x)$ satisfying the following two conditions:

- $h'(x) = 8 + x^3$
- The minimum value of $h(x)$ is -10.

W11. Find the function f with derivative $f'(x) = e^{2x}$ that passes through the point $P = (0, 9/2)$.

W12. Compute the following derivative:

$$\frac{d}{dt}\left(\tan^{-1}(te^t)\right)$$

Note: if you have trouble with this problem, I recommend looking at the resources linked under Homework 1 related to Inverse Trig Derivatives, the Chain Rule, and the Product Rule, and working through some of the example problems there.

W13. Consider the function

$$f(x) = \frac{5}{x^3} - \frac{7}{x^8}.$$

Let $F(x)$ be the antiderivative of $f(x)$ with $F(1) = 0$. Find $F(2)$.

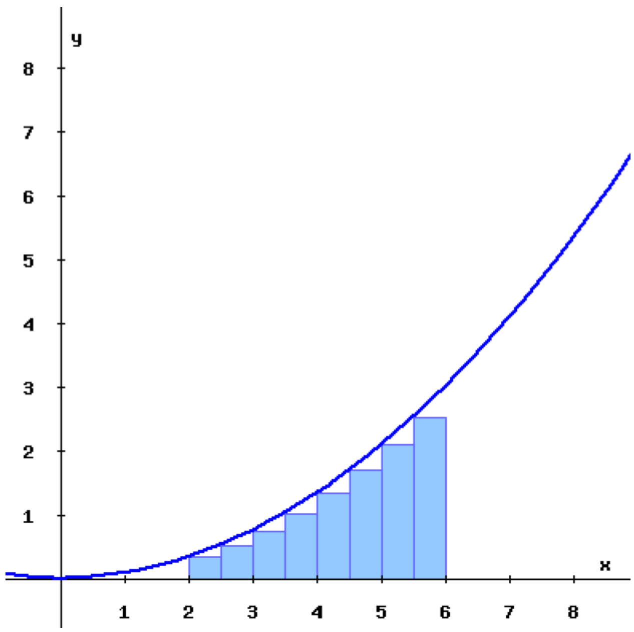
W14. Rewrite the following expression using summation notation:

$$\frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \frac{1}{32}x^5 + \frac{1}{64}x^6$$

W15. The rectangles in the graph below illustrate a left-endpoint Riemann sum for $f(x) = \frac{x^2}{12}$ on the interval $[2, 6]$.

(a) What is the value of this left-endpoint Riemann sum?

(b) Is this Riemann sum an overestimate or an underestimate for the area under the curve $y = f(x)$ on the interval $[2, 6]$?



W16. Write the integral

$$\int_2^6 \frac{x}{1+x^5}$$

as a limit of Riemann sums

(a) using right endpoints.

(b) using left endpoints.

W17. If $\displaystyle\int_0^{\pi/2} \frac{\cos(\theta)}{2-\sin(2\theta)}\,d\theta = \pi/4$, then what is $\displaystyle\int_0^{\pi/2} \frac{\cos(u)}{2-\sin(2u)}\,du$?

W18.

(a) Identify functions f and g where $f(g(\theta)) = \cos^3(\theta)$.

(b) Identify functions f and g where $f(g(w)) = e^{w^2-w-1}$.

Note: *if you would like a refresher, here is a video on identifying the parts of a composite function: https://bit.ly/3fxkj8s*

W19. Find two possible functions f such that $f''(x) = e^{-x}$.

W20. Let $f(x) = \int_0^x (t^3 + 7t^2 + 1)\, dt$. Find $f''(x)$.

W21. Let $f(s) = \int_0^s \frac{-3t+12}{1+\cos^2(t)}\, dt$. At what value of s does $f(s)$ have a local max?

W22.

(a) Evaluate the following definite integral to find $F(x)$. (Your answer will be a function of x.)

$$F(x) = \int_2^x \left(4t^3 - 6t^2 + 1\right)\, dt$$

(b) Find $F'(x)$ using two methods: first, by differentiating your answer from part (a). Second, using the Fundamental Theorem of Calculus, Part 1.

(c) Graph both $F(x)$ and $F'(x)$ (you can use a computer). Label a few points on both graphs, and briefly describe the geometric relationship between the two graphs at those points (a few words for each point is okay).

Note: part(c) will not be graded for accuracy, so don't worry about the exact form of your answer – just try to understand the relationship between the graphs!

W23. A biologist studies a population of bacteria. At the beginning there are 10 bacteria, and the growth rate of the population is $e^{t/6}$ bacteria per minute after t minutes. Find the number of bacteria after an hour (round your answer to the nearest whole number).

W24. Evaluate the following definite integral. $\int_{-\pi/2}^{\pi/2} |\sin(\theta)|\, d\theta$

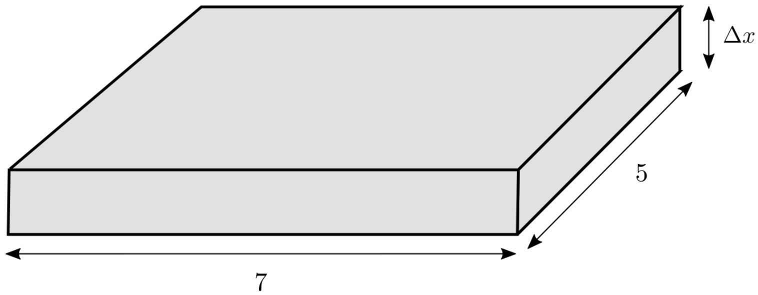
W25. Evaluate the following definite integral. $\int_{-1}^{1} x^3 \cos(x)\, dx$

W26. Evaluate the following definite integral. $\int_0^1 \frac{x}{\sqrt{x+1}}$

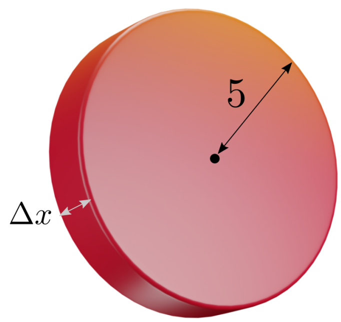
W27. Evaluate the following definite integral. $\int_0^1 \frac{x}{\sqrt{x+1}}$

W28. Evaluate the following definite integral. $\int_0^1 \frac{x}{\sqrt{x+1}}$

W29. Find the volume of the rectangular prism with dimensions listed below.



W30. Find the volume of a cylinder of radius 5 and height Δx.



W31. A 45 rpm vinyl record has radius 3.5 in, and has a central hole of radius .75 in. The thickness of the record is .025 in. What is the total volume of the record?

Photo of the Johnny Cash record "Folsom Prison Blues" by Daniel Hartwig is licensed under CC BY 2.0

W32. Evaluate the following definite integral. $\displaystyle\int_0^1 \frac{x}{\sqrt{x+1}}\,dx$

W33. Evaluate the following integral. $\displaystyle\int_0^{\pi/3} \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$

W34. Evaluate the following integral. $\displaystyle\int \frac{\tan^{-1}(2t)}{1+4t^2}\,dt$

W35. Evaluate the following integral. $\displaystyle\int \frac{e^t}{\sqrt{1-e^{2t}}}\,dt$

W36. Evaluate the following integral. $\displaystyle\int_0^{1/2} \frac{\tan\left(\sin^{-1} t\right)}{\sqrt{1-t^2}}\,dt$

W37. Evaluate the following integral. $\displaystyle\int_{1/50}^{2} \frac{e^{\frac{2}{w}}}{w^2}\,dw$

W38. Evaluate the following integral. $\displaystyle\int_0^{\ln(1+\pi)} e^x \cos(1-e^x)\,dx$

W39. Evaluate the following integral. $\displaystyle\int \cos^3\theta\,d\theta$ *(Hint:* $\cos^2\theta = 1-\sin^2\theta$*)*

W40. ***Note:*** *the next few problems are algebra review to prepare for an integration technique called "partial fractions." In this method, we use algebra to break rational functions into simpler pieces, which can be integrated using techniques we've already learned (e.g. u-substitution). If you'd like to practice/preview the method itself before seeing it in class, here is a tutorial on partial fractions (no calculus, just algebra): http://bit.ly/3sFYeHZ*

Factor each of the following polynomials, or determine that they are irreducible:

(a) $x^2 - 1$

(b) $x^2 + 1$

(c) $x^2 - 6x + 9$

If you would like more practice/review factoring, here is a resource: http://bit.ly/3DTuo9H

W41. Factor each of the following polynomials, or determine that they are irreducible:

(a) $x^3 - x^2 - 2x$

(b) $x^3 + 4x$

W42. In parts (a) and (b) below, combine the terms into a single rational expression.

(a) $\dfrac{1/2}{x-1} - \dfrac{1/2}{x+1}$

(b) $\dfrac{1}{x-2} - \dfrac{4}{(x+2)^2} - \dfrac{1}{x+2}$ *(you can leave the denominator factored – it gets messy otherwise)*

Here is a review resource on working with rational expressions: http://bit.ly/3Nk9jrJ (Example 3 is especially relevant).

W43. Rewrite each of the following rational expressions as a quotient and a remainder, using polynomial long-division or another method of your choice (e.g. synthetic division).

(a) $\dfrac{3x^2+5x-1}{x+2}$

(b) $\dfrac{x^4-1}{x-1}$

Here is a review resource on polynomial long-division: http://bit.ly/3TUtTl5

W44. Evaluate the following integral. $\displaystyle\int \sin^2\theta \, d\theta$

Hint: $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$. *This "power-lowering" identity comes from the double-angle formula for cosine. Although we won't use trig identities very much in this class, if you're interested, here is a quick and clean review of the most useful ones for calculus: http://bit.ly/3sKP2SV*

W45. A certain bacteria population P obeys the exponential growth law

$$P(t) = 1500e^{2.1t}$$

(t in hours).

(a) How many bacteria are present initially?

(b) At what time will there be 10,000 bacteria?

W46. The decay constant of Polonium-84 is $k = 0.0067$ *(the units for k are "inverse years," i.e., "per year")*. What is the half-life of Polonium-84?

Note: *there was a typo in the original problem statement for W46, which has now been fixed; but if you did the original version, that's okay – you don't need to redo it!*

W47. Newton's Law of Cooling has applications in a wide variety of fields, including forensics. For example: police arrive at the scene of a murder at 12am, and measure the temperature of the body, finding that it is 90°F. At 1:30am, after further inspection of the crime scene, the body's temperature has dropped to 87°F. The temperature of the crime scene has remained constant at 82°F. Assuming the victim had a normal body temperature of 98.6°F at the time of death, approximately what time did the victim die?

W48. The population of a city is

$$P(t) = 9e^{0.17t}$$

(in millions), where t is measured in years.

(a) Find the doubling time of the population.

(b) How long does it take for the population to triple in size?

W49. Evaluate the following integral. $\int e^{\theta} \cos\theta \, d\theta$

If you get stuck, there is a solution to this problem here (Example 8): http://bit.ly/3FOzh54
I'd encourage you to really try the problem before looking, and if you're having trouble, only read a tiny bit at a time to get you unstuck. (Or read it after completing the problem to check your work.)

W50. Evaluate the following integral. $\int t^7 \sin(2t^4)\, dt$

If you get stuck, there is a solution here ("Practice Problems" tab, #7): http://bit.ly/3FOzh54
I'd encourage you to really try the problem before looking, and if you're having trouble, only read a tiny bit at a time to get you unstuck. (And in any case, I encourage you to read the whole solution after finishing the problem! This solution discusses the strategy of the problem, and shares a couple of valid approaches – it can be really helpful to compare different techniques!)

W51. Use the method of partial fractions to break down the following rational expression. $\dfrac{x^2+3x+1}{x^2-4}$

W52. Evaluate the following integral. If the integral is not convergent, answer "divergent." $\displaystyle\int_0^{\infty} e^{-5x}\, dx$

W53. Evaluate the following integral. If the integral is not convergent, answer "divergent." $\displaystyle\int_{-1}^{1} \frac{1}{x^{2/3}}\, dx$

W54. An 8-kg quantity of a radioactive isotope decays to 1 kg after 2 years. Find the decay constant k of the isotope.

W55. *This problem is meant as preparation for evaluating improper integrals, where the final step is usually to evaluate a limit like these. For a refresher and some additional practice problems, here is a review resource on limits at infinity and infinite limits: http://bit.ly/3E2apUV (Paul's Notes is also good for this!)*

Evaluate the following limits if they exist. (If the limit is ∞ or $-\infty$, say so.)

(a) $\displaystyle\lim_{t\to\infty} \left(e^{-t^2} - 1\right)$

(b) $\displaystyle\lim_{t\to\infty} \sin(t)$

(c) $\lim_{t \to 0^-} \frac{1}{t^3}$

W56. *This problem is meant as preparation for using the comparison test for improper integrals. To successfully use the comparison test, the first step is always to identify the "dominant" or "most important" terms in the function being integrated. Although this step doesn't always have to be formally written down, it is often helpful in choosing a successful strategy.*

Consider the following functions, where $x > 0$:

- x^4
- $\ln x$
- $\sqrt{x}$
- $x - 5 + \sin x$
- e^x

Rank the functions from **smallest** to **largest**:

(a) when x is **large**.

(b) when x is **close to zero**.

You may just be able to "see" some of these. But for any that you find more subtle, here is a tutorial on comparing the growth rate of functions using limits: http://bit.ly/3toSh2H

W57. Evaluate the integral. $\int e^{-2t} \cos t \, dt$

W58. Evaluate the integral. $\int (3x^2 + 1) \tan^{-1}(x) \, dx$

W59. Evaluate the integral. $\int e^{3\sqrt{r}} \, dr$

W60. Evaluate the integral. $\int \frac{8}{(x-4)^2(x+4)} \, dx$

W61. Evaluate the integral. $\int \frac{x^2 - 2x + 3}{(x-2)(x^2+1)} \, dx$

W62. Find the area under the curve $y = 3xe^{-x}$ for $x \geq 0$.

W63. (a) For which numbers $p > 0$ does the following integral converge, and for which does it diverge?

$$\int_1^\infty \frac{1}{x^p}$$

(Hint: if you can't figure out where to start, try seeing what happens when you evaluate the improper integral for $p = \frac{1}{2}$, $p = 1$, and $p = 2$.)

(b) For which numbers $p > 0$ does the following integral converge, and for which does it diverge?

$$\int_0^1 \frac{1}{x^p}$$

(c) Graph the function $\frac{1}{x^p}$ for several different values of p (you can use a computer). Can you explain how the different graphs are consistent with your answers in parts (a) and (b)?

The answers to parts (a) and (b) of this problem are sometimes called the "p-test for improper integrals," and these integrals are often useful when we want to use the comparison test (they give us a family of simple examples to compare to).

W64. Use the comparison test (comparing with an appropriate integral) to determine whether the following integral converges or diverges.

$$\int_0^1 \frac{\cos^2(x)}{\sqrt{x}}\,dx$$

W65. Use the comparison test (comparing with an appropriate integral) to determine whether the following integral converges or diverges.

$$\int_1^\infty \frac{1}{x^2+5-\cos x}\,dx$$

W66. Don't forget to fill out CAPES!

Link for CAPEs: `https://cape.ucsd.edu/students/`
Link for TA evaluations (I think): `http://academicaffairs.ucsd.edu/Modules/Evals`

This is "honor system" – there is nothing you need to submit for this question! But if you want, you can submit a screenshot of the confirmation page after you submit your CAPEs, or a meme of your choice :)

W67. Evaluate the following sum: $\displaystyle\sum_{n=1}^{4} \frac{1}{3^{n-1}}$

W68. An airplane starts at a cruising altitude of 40,000 ft above sea level, and its altitude changes at a rate of $f(t)$ ft/s.

(a) What is represented by the quantity $40,000 + \int_0^{60} f(t)\,dt$?

(b) If $\int_0^{120} f(t)\,dt = -45,000$, what must have happened to the plane?

W69. Evaluate the indefinite integral

$$\int \cos^2(x)\sin^3(x)\,dx$$

Hint: $\sin^2(x) = 1 - \cos^2(x)$. *You can use this to replace some of the factors of* $\sin x$*, then do a bit of algebra.*

W70. Use the comparison test (comparing with an appropriate integral) to determine whether the following integral converges or diverges.

$$\int_e^\infty \frac{\ln x}{x}\, dx$$

W71. Draw (by hand) the direction field for the differential equation $y' = xy$. Include the points where $x = -1, 0$, or 1 and $y = -1, 0$, or 1 (so your graph should have 9 points in total).

W72. (a) Use a computer to draw the direction field for the differential equation $y' = y^2 - 2y$, as well as the particular solutions passing through the points $(0, 1)$ and $(0, -1)$. *Here is an online tool you can use for this: https://www.geogebra.org/m/W7dAdgqc*

(b) Judging from the direction field, does it look the differential equation has any equilibrium solutions? Confirm your answers by double-checking that they satisfy the differential equation.

W73. Find the limit of the sequence or show that the sequence diverges.

$$a_n = 20 - \frac{2}{n^3}$$

W74. Does the following geometric series converge or diverge? If it does converge, find its value.

$$\sum_{n=1}^{\infty} \frac{7^n}{14^n}$$

W75. Does the following geometric series converge or diverge? If it does converge, find its value.

$$10 + \frac{10e}{\pi} + \frac{10e^2}{\pi^2} + \frac{10e^3}{\pi^3} + \frac{10e^4}{\pi^4} + \cdots$$

W76. Use a geometric series to write 0.777777.... as a fraction of integers.