Math 10B Website Problems

Answer Key

W1. 11

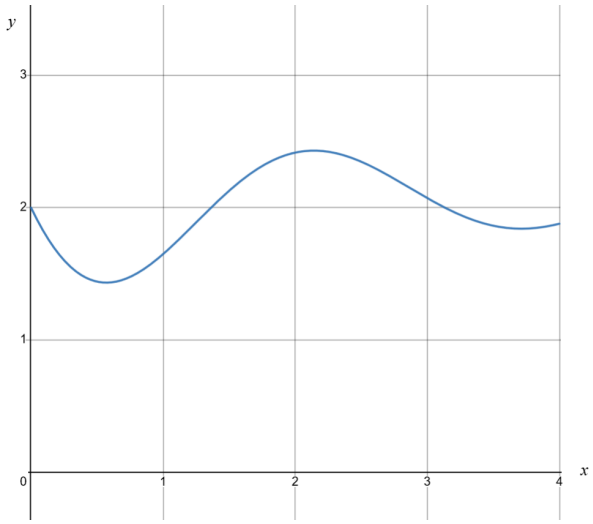
W2. 8π

W3. Yes: $f(x)$ is continuous on $[2,5]$ and differentiable on $(2,5)$ (polynomial, so continuous and differentiable everywhere). $c = 7/2$.

W4. 30

W5. $9x^8 + 16x^{11} + 25x^{14}$

W6.



W7. $F(t) = -10\sin(t) - 5\cos(t) + C$ (where C can represent *any* constant)

W8. $F(x) = 10\sin(x) + 2\cos(x) - 1$

W9. $R = -70t^{-1} + 80$

W10. $h(x) = 8x + \frac{1}{4}x^4 + 2$

W11. $f(x) = \frac{1}{2}e^{2x} + 4$

W12. $\dfrac{(1+t)e^t}{t^2e^{2t}+1}$

W13. $F(2) = \frac{113}{2^7} \approx .88$

W14. $\displaystyle\sum_{j=1}^{6} 2^{-j}x^j$

W15.

(a) $\frac{41}{8} = 5.125$

(b) underestimate

W16. (a) $\displaystyle\int_2^6 \frac{x}{1+x^5} = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(\frac{4}{n}\cdot\sum_{i=1}^{n}\frac{\left(2+4\frac{i}{n}\right)}{1+\left(2+4\frac{i}{n}\right)^5}\right)$

(b) $\displaystyle\int_2^6 \frac{x}{1+x^5} = \lim_{n\to\infty} L_n = \lim_{n\to\infty}\left(\frac{4}{n}\cdot\sum_{i=1}^{n}\frac{\left(2+4\frac{i-1}{n}\right)}{1+\left(2+4\frac{i-1}{n}\right)^5}\right)$

W17. $\pi/4$

W18.

(a) The simplest correct answer is $f(\theta) = \theta^3$ and $g(\theta) = \cos(\theta)$.

(b) The simplest correct answer is $f(w) = e^w$ and $g(w) = w^2 - w - 1$.

W19. Answers may vary: two possible answers are $f(x) = e^{-x} + 5$ and $f(x) = e^{-x} - 3x$. All answers are of the form $f(x) = e^{-x} + Cx + D$, where C and D are constants.

W20. $f''(x) = 3x^2 + 14x$.

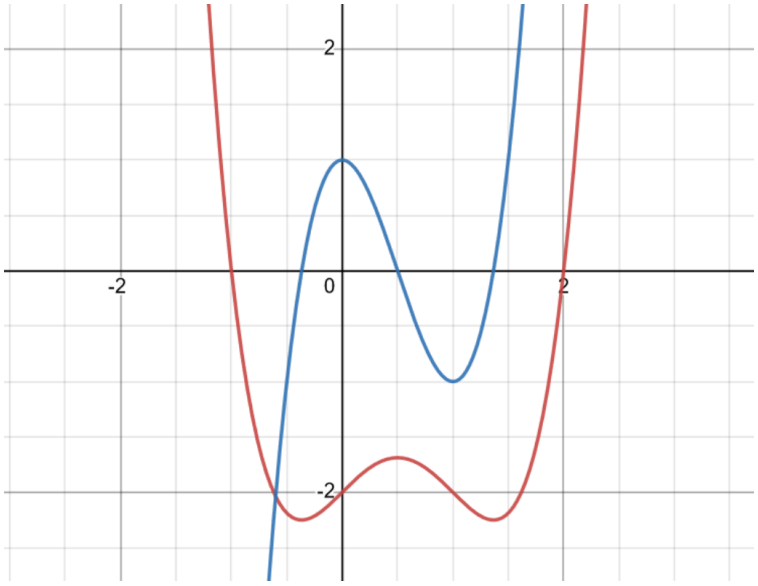
W21. $t = 4$.

W22.

(a) $F(x) = x^4 - 2x^3 + x - 2$

(b) With both methods, we obtain $F'(x) = 4x^3 - 6x^2 + 1$.

(c) Answers will vary. Here are the graphs, with $F(x)$ in red and $F'(x)$ in blue.



W23. $6e^{10} + 4 \approx 132,163$ bacteria

W24. 2

W25. 0

W26. $-\frac{2}{3}(\sqrt{2} - 2)$

W27. $-\frac{2}{3}(\sqrt{2} - 2)$

W28. $-\frac{2}{3}(\sqrt{2} - 2)$

W29. $35 \cdot \Delta x$

W30. $25\pi \cdot \Delta x$

W31. $.2921875\pi$ in$^3 \approx .92$ in^3

W32. $-\frac{2}{3}(\sqrt{2} - 2)$

W33. $\ln\left(\sqrt{3} - 1\right)$

W34. $\frac{1}{4}\left(\tan^{-1}(2t)\right)^2 + C$

W35. $\sin^{-1}(e^t) + C$

W36. $\frac{1}{2}\ln\left(\frac{4}{3}\right)$

W37. $\frac{1}{2}\left(e^{100}-e\right)$

W38. 0

W39. $\sin\theta - \frac{1}{3}\sin^3\theta + C$

W40.

(a) $(x+1)(x-1)$

(b) Irreducible (can check: quadratic formula gives imaginary roots)

(c) $(x-3)^2$

W41.

(a) $x(x-2)(x+1)$

(b) $x(x^2+4)$ *(Cannot be factored further: x^2+4 is an irreducible quadratic)*

W42.

(a) $\dfrac{1}{x^2-1}$

(b) $\dfrac{16}{(x+2)^2(x-2)}$

W43.

(a) $3x-1+\dfrac{1}{x+2}$

(b) x^3+x^2+x+1

W44. $\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta) + C$

W45.

(a) 1500 bacteria

(b) $t = \frac{\ln(10000/1500)}{2.1} \approx 0.90339$ hours

W46. $\frac{\ln(2)}{.0067} \approx 103.45$ years.

W47. If the time t is measured in hours starting at 12am (so 12am is $t=0$), then using Newton's Law of Cooling, the time of death is $t = -1.5\ln(16.6/8)/\ln(8/5) \approx -2.33$, about 9:40pm.

W48. (a) $\frac{\ln(2)}{0.17} \approx 4.077$ years

(b) $\frac{\ln(3)}{0.17} \approx 6.462$ years

W49. $\frac{1}{2}\left(e^{\theta}\cos\theta + e^{\theta}\sin\theta\right) + C$

W50. $-\frac{1}{8}t^4\cos(2t^4) + \frac{1}{16}\sin(2t^4) + C$

W51. $1 + \frac{11/4}{x-2} + \frac{1/4}{x+2}$

W52. $\frac{1}{5}$

W53. 6

W54. $k = \frac{\ln(1/8)}{-2} \approx 1.0397/\text{yr}$

W55. (a) -1

(b) Does not exist

(c) $-\infty$

W56.

(a) $\ln x < \sqrt{x} < x - 5 + \sin x < x^4 < e^x$ when x is large.

(b) $\ln x < x - 5 + \sin x < x^4 < \sqrt{x} < e^x$ when x is close to zero (and $x > 0$).

W57. $\frac{1}{5}e^{-2t}\sin t - \frac{2}{5}e^{-2t}\cos t + C$

W58. $(x^3 + x)\tan^{-1}(x) - \frac{1}{2}x^2 + C$

W59. $\frac{2}{3}\sqrt{r}\cdot e^{3\sqrt{r}} - \frac{2}{9}e^{3\sqrt{r}} + C$

W60. $-\frac{1}{x-4} - \frac{1}{8}\ln|x-4| + \frac{1}{8}\ln|x+4| + C$

W61. $\frac{1}{5}\ln|x^2+1| + \frac{3}{5}\ln|x-2| - \frac{6}{5}\tan^{-1}(x) + C$

W62. 3

W63. (a) The integral converges for $p > 1$ and diverges for $p \leq 1$.

(b) The integral converges for $p < 1$ and diverges for $p \geq 1$.

(c) (answers will vary)

W64. Converges

W65. Converges

W66. Answers will vary :)

W67. $\frac{40}{27} \approx 1.48$

W68.

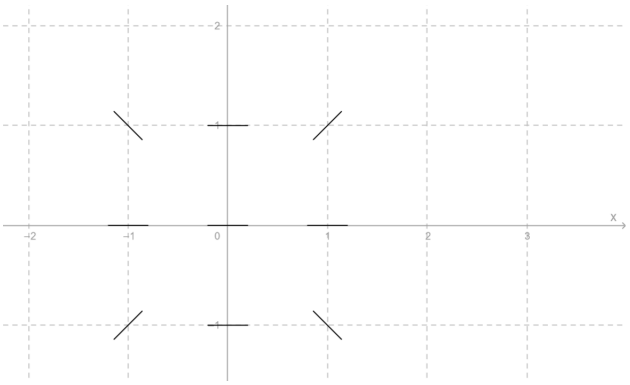
(a) The altitude of the airplane after 1 minute (60 seconds).

(b) It crashed and sank :(It started at 40,000 feet above sea level, and after 2 minutes, it had lost 45,000 feet of altitude.

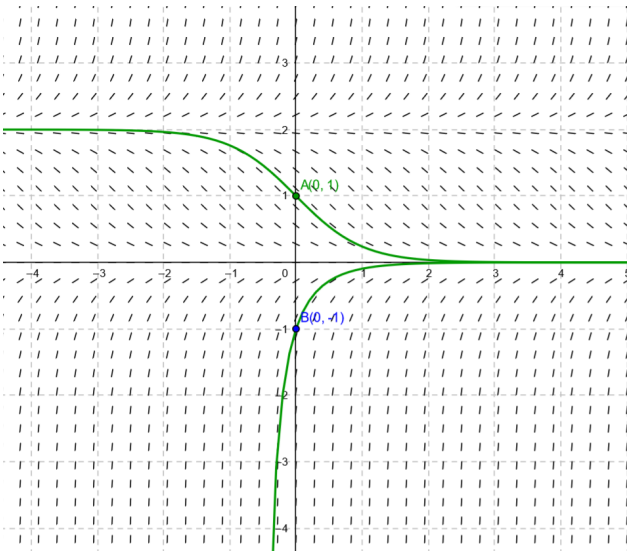
W69. $\frac{1}{5}\cos^5(x) - \frac{1}{3}\cos^3(x) + C$

W70. Diverges

W71.



W72.



(a)

(b) Yes, $y = 0$ and $y = 2$.

W73. 20

W74. Converges to 1

W75. Converges to $\dfrac{10\pi}{\pi - e}$

W76. $\frac{7}{9}$