Nov 18, 2013

$P(Q/R)$ probability that Q occurs and not R.
$D(Q,R)$ probability that exactly 1 of Q and R occur.

<u>Bell's inequality</u>

$$P(Q/R) + P(R/S) \geq P(Q/S)$$
$$\Rightarrow P(Q/R_1) + P(R_1/R_2) + \cdots + P(R_{n-1}/R_n) + P(R_n/S) \geq P(Q/S)$$

$A \leftarrow \bullet \rightarrow B$

A and B equally polarized photons with either a polarization or its opposite

$$\Psi = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$= \frac{1}{\sqrt{2}}(|\theta\theta\rangle + |\tfrac{\pi}{2}-\theta, \tfrac{\pi}{2}, \theta\rangle)$$

Since $(\rho(\theta) \otimes \rho(\theta))\Psi = \Psi$ $\rho(\theta)$ rotation through θ.

A: particle with polarization θ, probability $P(Q)$

we have a filter S polarizes to ϕ. $P(S)$ the probability

particle probability

$$\langle(\cos\theta, \sin\theta) | (\cos\phi, \sin\phi)\rangle^2$$
$$= (\cos\theta\cos\phi + \sin\theta\sin\phi)^2$$
$$= (\cos(\theta-\phi))^2$$

Thus $P(Q/S) = P(Q)(1 - \cos(\theta-\phi)^2)$
$$= P(Q)\sin(\theta-\phi)^2$$

A: photon

q, s: filters of add an angle of ϕ.

Q_A, S_A: instances of these filters. Similarly for B

r t all filters add ϕ to angle

R_B T_B

$$P(Q_A/R_B) + P(R_B/S_A) + P(S_A/T_B) - P(Q_A/T_B)$$

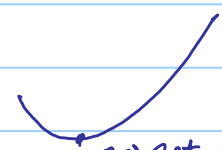
want to show this is negative.

$$= P(Q_B/R_B) + P(R_B/S_B) + P(S_B/T_B) - P(Q_B/T_B)$$ { since if A is polarized θ so is B

$$= \frac{1}{2}(3\sin^2\phi - \sin^2 3\phi) =: f(\phi)$$ (since all polarizations are equally probable and are $\frac{1}{2}$)

if $\phi = \frac{\pi}{6}$, get $-\frac{1}{8}$

$$f(\phi) = \frac{1}{2}(3\phi^2 + O(\phi^3) - 9\phi^2 + O(\phi^3))$$
$$= \frac{1}{2}((3-9)\phi^2) + O(\phi^3)$$



$\frac{\pi}{8} \rightsquigarrow$ get $\frac{1}{2}(1-\sqrt{2})$

Exercise ($\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}$)

Hint: Use

Observation:

① Classical probability is violated in quantum mechanics (Entanglement is critical)

② locality is violated.

EPR ↓

GHZ

What follows is a method of using the GHZ state to demonstrate another paradoxical quantum mechanical effect.

We look at 3 independant spin $\frac{1}{2}$ states yielding the state

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) = GHZ$$

in 3-qubits.

We consider in 1-qubit that

Two observables X & Y. Take values ± 1 so

$$X^2 = Y^2 = I$$

Suppose that an Observation of 3 with two of the factors having a Y is always -1.

$X_1 \otimes Y_2 \otimes Y_3$ eigenvalue -1

$Y_1 \otimes X_2 \otimes Y_3$

$Y_1 \otimes Y_2 \otimes X_3$

Then

$$(X_1 Y_2 Y_3)(Y_1 X_2 Y_3)(Y_1 Y_2 X_3) \to -1$$

$$(X_1 Y_1 Y_1)(Y_2 X_2 Y_2)(Y_3 Y_3 X_3).$$ If Y, X commute this is $X_1 X_2 X_3$ since $Y^2 = I$.

Quantum measurement: $X = \sigma_1$, $Y = \sigma_2$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_1 |0\rangle = |1\rangle, \quad \sigma_1 |1\rangle = |0\rangle$$

$$\sigma_2 |0\rangle = i|1\rangle, \quad \sigma_2 |1\rangle = -i|0\rangle$$

$$X_1 Y_2 Y_3 |GHZ\rangle = (\sigma_1 \otimes \sigma_2 \otimes \sigma_2)(|0\rangle \otimes |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle \otimes |1\rangle)/\sqrt{2}$$

$$= -GHZ.$$

Satisfies the condition

$$(X_1 Y_2 Y_3)(Y_1 X_2 Y_3)(Y_1 Y_2 X_3) GHZ = -GHZ$$

Thus any of the three give $-GHZ$ so

$$\underbrace{(X_1 Y_1 Y_1)}_{\substack{I \\ \| \\ X_1}} (Y_2 X_2 Y_2) \underbrace{(Y_3 Y_3 X_3)}_{\substack{I \\ \| \\ X_3}} |GHZ\rangle$$

$$\sigma_1 \sigma_2 + \sigma_2 \sigma_1 = 0$$

$$Y_2 X_2 = -X_2 Y_2$$

$$= -X_1 X_2 X_3 |GHZ\rangle$$

This gives $X_1 X_2 X_3 |GHZ\rangle = GHZ$

classically should get $-GHZ$

Exercise: Show $\nexists\, X, Y$ for $|000\rangle$

$$\frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$ state

Can you find X, Y for this?