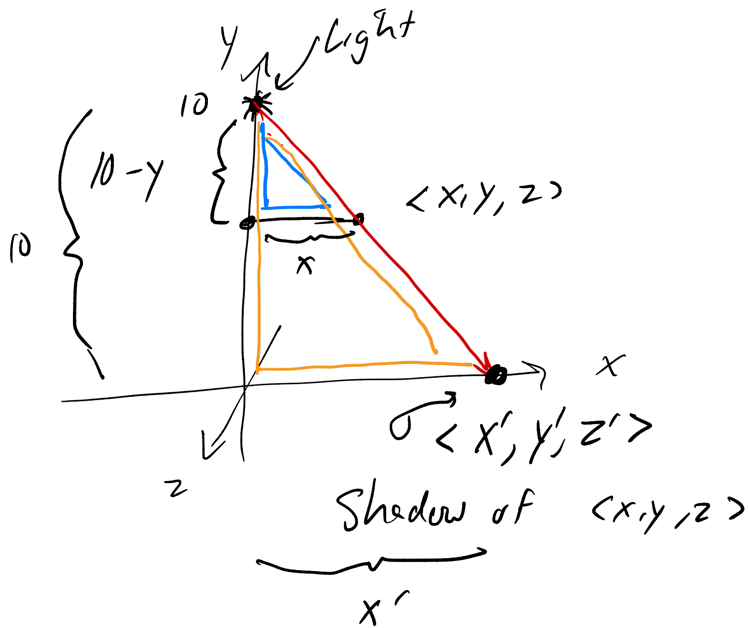
Shadows (via perspective-style transformations)

Example: Light source at $\langle 0, 10, 0 \rangle$, casts shadows onto the xz-plane



Formula for x', y', z':

$$y' = 0 \quad \checkmark$$

To find x', use similar triangles (after orthogonally projecting onto the xy-plane.

$$\frac{x'}{10} = \frac{x}{10-y} \quad \text{so} \quad x' = \frac{10x}{10-y}$$

Similarly, $z' = \dfrac{10z}{10-y}$

Let's express with a 4x4 matrix....

$$\langle x, y, z\rangle \longmapsto \left\langle \frac{10x}{10-y}, 0, \frac{10z}{10-y} \right\rangle$$

In homogeneous coordinates

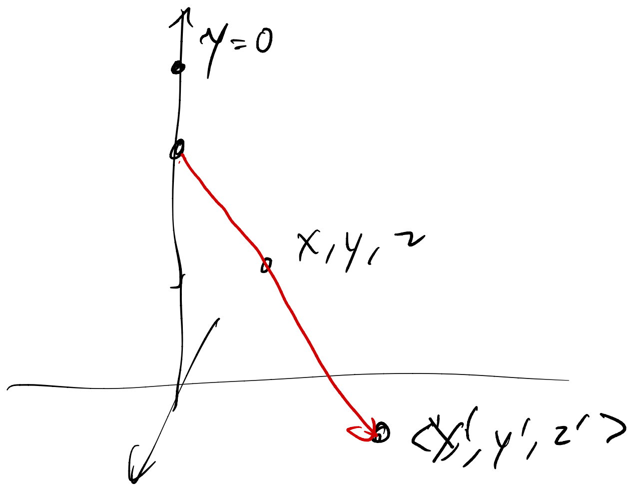
$$\langle x, y, z, 1\rangle \longmapsto \left\langle \frac{10x}{10-y}, 0 \frac{10z}{10-y}, 1 \right\rangle$$

Alternately:

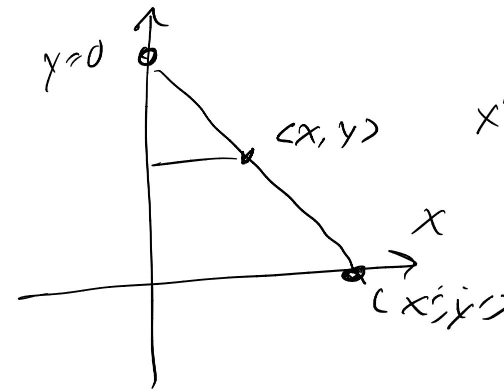
$$\langle x, y, z, 1\rangle \longmapsto \langle 10x, 0, 10z, 10-y\rangle$$

Represent the same point in $\mathbb{R}^3$.

$$\begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & -1 & 0 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} 10x \\ 0 \\ 10z \\ 10-y \end{pmatrix}$$

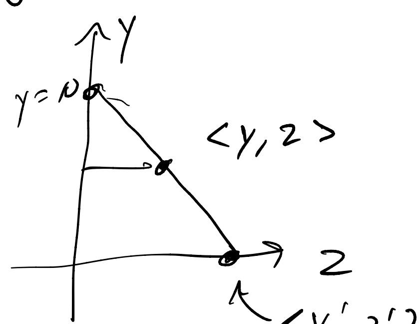


Project on xy plane



$$x' = \frac{10x}{10-y}$$

Project onto the yz-plane



$$z' = \frac{10z}{10-y}$$

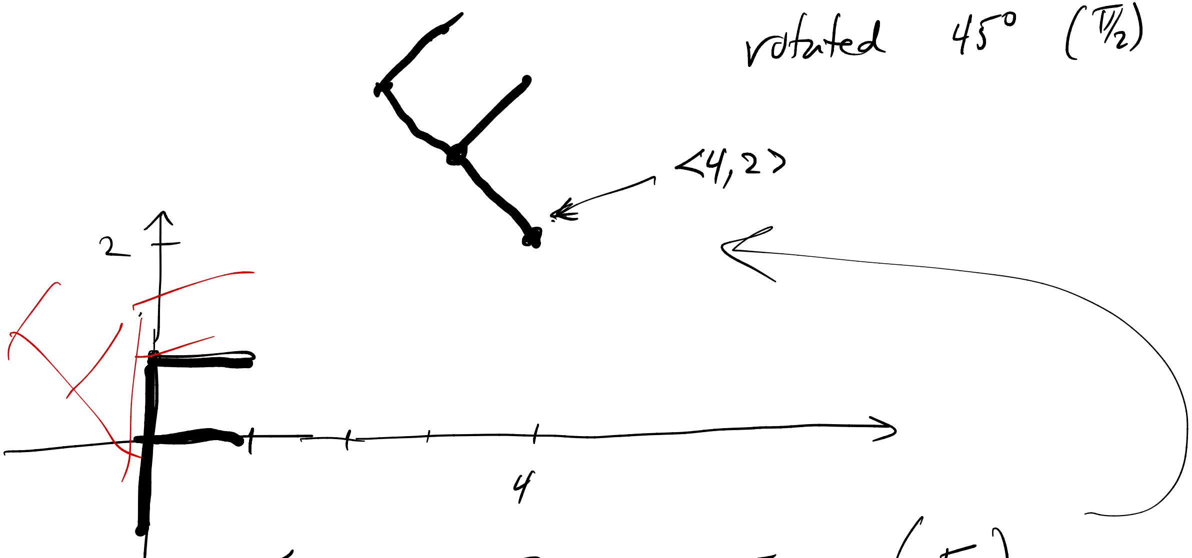
Another way to view composition:

Recall a composition $A_1 \circ A_2 \circ A_3 \circ \cdots \circ A_k$ acts on a $\boxed{\text{geometry}}$ by applying A_k, then A_{k-1}, ..., finally A_1.

We can also view them as acting in the order A_1, then A_2, then A_3, ..., but acting on a coordinate system.

Example:

rotated 45° $(\pi/2)$ ccw



$$T_{\langle 4,2\rangle} \circ R_{\pi/4} \circ T_{\langle 0,1\rangle}(F)$$

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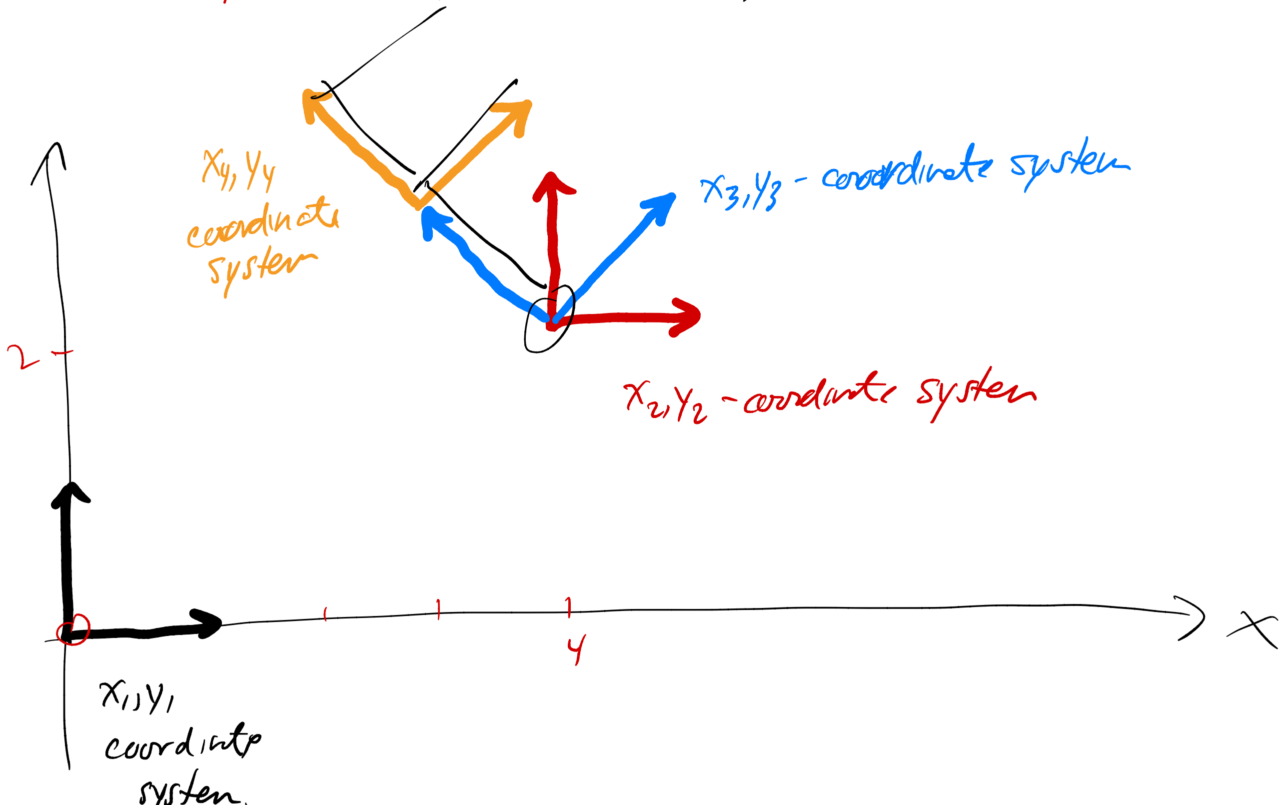
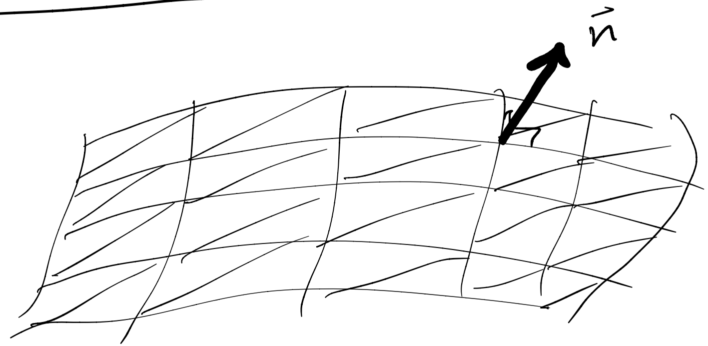


Surfaces + Normal Vectors:



Surface is rendered as a bunch of triangles

Specify vertices, how they form triangle and normal vectors at vertices

"Normal" = "Perpindicular" to the surface.

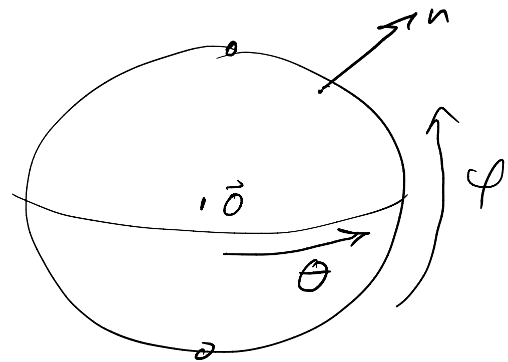
__Example__ Sphere - unit sphere centered at $\vec{0}$.

How to parameterize the spherical

Spherical Coordinates

$0 \le \theta \le 2\pi$

$\theta \approx$ how far gone around



$-\frac{\pi}{2} \le \varphi \le \pi/2$

South Pole — North pole

$$\vec{f}(\theta, \varphi) = \langle \sin\theta \cos\varphi,\ \sin\varphi,\ \cos\theta \cos\varphi \rangle$$

$\boxed{\text{"Parametric surface"}}$.

$$\vec{n}(\theta, \varphi) = \vec{f}(\theta, \varphi) = \langle \sin\theta \cos\varphi,\ \sin\varphi,\ \cos\theta \cos\varphi \rangle$$

- Also a function of θ, φ.

Unit Sphere $\mathcal{S} = \{ \langle x,y,z \rangle : x^2 + y^2 + z^2 = 1 \}$

$\boxed{\text{Level Set}}$, ie. $\mathcal{S} = \{ \langle x,y,z \rangle : \underbrace{x^2+y^2+z^2-1}_{h(x,y,z)} = 0 \}$

If $\langle x,y,z \rangle \in \mathcal{S}$, i.e if $h(x,y,z) = 0$,

then in this case, the normal

vector is $\vec{n} = \langle x,y,z \rangle$.

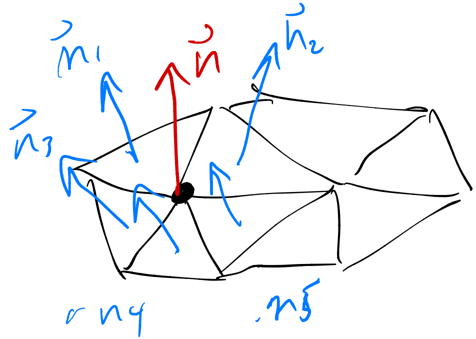
Now $\vec{n}$ - expressed in terms of x,y,z.

Method #1 for specifying a surface & normals

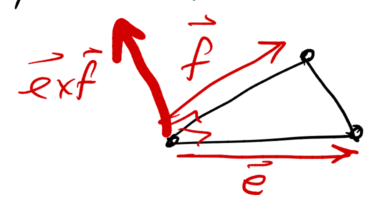
Given: surface as a bunch of (x, y, z) vertex values and a triangulation. — $\boxed{\text{Surface is approximated by triangles.}}$

Example A 3-D scanner

No mathematical formula for the surface.



Each triangle has a easy to compute normal.



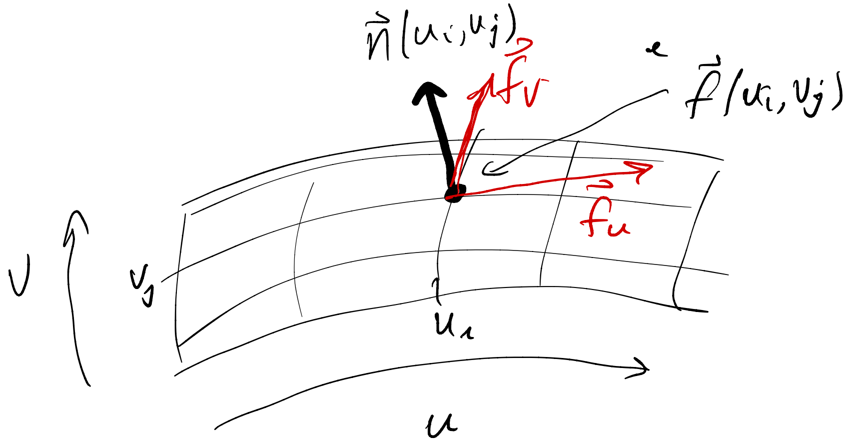
$$\frac{\vec{e}\times\vec{f}}{\|\vec{e}\times\vec{f}\|} = \vec{n}$$

The normal $\vec{n}$ at a vertex can be estimated as

$$\vec{n} = \left(\sum_i \vec{n}_i\right) \Big/ \left\|\sum_i \vec{n}_i\right\|.$$

$\vec{n}_i$'s are the normals of the triangles using that vertex

Method #2: Parametric Surface



$\vec{f}(u,v)$

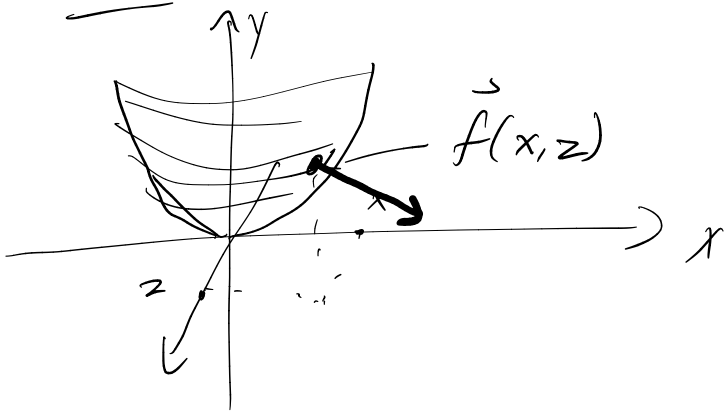
$$\vec{f}_u = \frac{\partial \vec{f}}{\partial u} = \lim_{\delta \to 0} \frac{\vec{f}(u_i+\delta, v_i) - \vec{f}(u_i, v_i)}{\delta}$$ is shown above and is tangent to the surface.

$$\vec{f}_v = \frac{\partial f}{\partial v}$$ as shown above.

Take $\vec{n} = \vec{f}_u \times \vec{f}_v$ or $(\vec{f}_u \times \vec{f}_v) / \| \vec{f}_u \times \vec{f}_v \|$

Note $\vec{n} = \vec{n}(u,v)$

Example Paraboloid $\qquad y = x^2 + z^2$



Take u, v equal to x, z

$$\vec f(x,z) = \langle x, x^2+z^2, z \rangle$$

$$\vec f_x = \langle 1, 2x, 0 \rangle$$

$$\vec f_z = \langle 0, 2z, 1 \rangle$$

$$\vec f_x \times \vec f_y = \langle 2x, -1, 2z \rangle$$

(Watch out for the sign!)