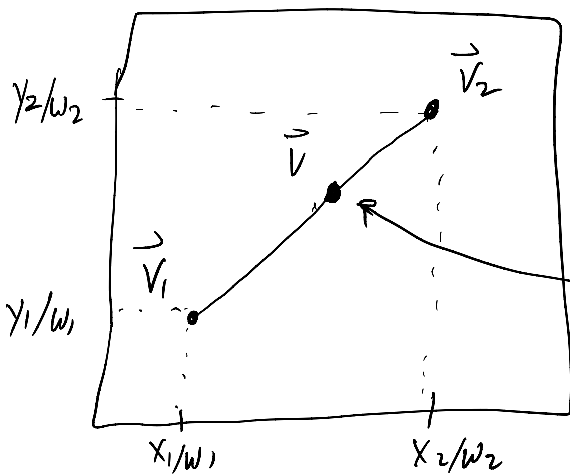
Hyperbolic interpolation:

Basic issue

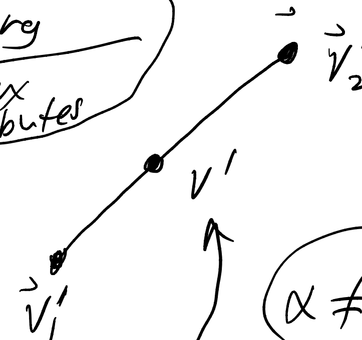
Line in space maps to the line on the screen.



Screen

Depth values of $\vec{v}$ is obtained by lerping with β.
Not so for vertex attributes

$\vec{v} = \mathrm{lerp}(\vec{v}_1, \vec{v}_2, \beta)$



$v' = \mathrm{lerp}(v_1', v_2', \alpha)$

$\alpha \neq \beta$

$\mathbb{R}^3$

$\vec{u}_1 = \langle x_1, y_1, z_1, w_1 \rangle$

- output by Fragment Shade

$u_2 = \langle x_2, y_2, z_2 w_2 \rangle$

$\vec{v}_1 = \langle x_1/w_1 \rangle$

$\vec{v}_2 = \langle x_2/w_2 \rangle$

$\vec{v}_1' \longmapsto \vec{v}_1$

$v_2' \longmapsto \vec{v}_2$

$v' \longmapsto \vec{v}$

<u>Shift gears</u>: What does lerp-ing do to homogeneous coordinates?

<u>Example</u> $\vec{x} = \langle 0,0 \rangle \in \mathbb{R}^2$
$\vec{y} = \langle 2,0 \rangle \in \mathbb{R}^2$

$\vec{u} = \langle 0,0,1 \rangle$ represents $\vec{x}$
$\vec{v} = \langle 2,0,1 \rangle$ " $\vec{y}$
$\vec{w} = \langle 4,0,2 \rangle$ also represents $\vec{y}$

$\text{lerp}(\vec{x},\vec{y},\tfrac{1}{2}) = \langle 1,0 \rangle.$

$\text{lerp}(\vec{u},\vec{v},\tfrac{1}{2}) = \langle 1,0,1 \rangle$ represents $\langle 1,0 \rangle$ ✓

$\text{lerp}(\vec{u},\vec{w},\tfrac{1}{2}) = \langle 2, 0, \tfrac{3}{2} \rangle$ represents $\langle \tfrac{4}{3},0 \rangle \neq \langle 1,0 \rangle$

$\text{lerp}(\vec{u},\vec{w},\tfrac{1}{2}) = \text{lerp}(\vec{x},\vec{y},\tfrac{2}{3}) = \tfrac{1}{3}\vec{x} + \tfrac{2}{3}\vec{y}.$

$\vec{w}$ - with 3rd component 2 is <u>weighted</u> twice as much as $\vec{u}$ with 3rd component 1.

More generally Let $\vec{x} \in \mathbb{R}^n$. Suppose $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$

Write $\langle \vec{x}; w \rangle$ for $\langle x_1 \ldots x_k, w \rangle$

So $\langle w\vec{x}; w \rangle$ is a homogeneous representation for $\vec{x}$.

Now Let $\vec{y}_1, \ldots, \vec{y}_k \in \mathbb{R}^n$. Let $w_1 \ldots w_k > 0$

Let $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathbb{R}$ $\alpha_1 + \alpha_2 + \ldots + \alpha_k = 1$

Form affine combination

$$\vec{z} = \boxed{\alpha_1 \langle w_1\vec{y}_1; w_1 \rangle + \alpha_2 \langle w_2 \vec{y}_2, w_2 \rangle + \ldots + \alpha_k \langle w_k \vec{y}_k, \vec{y}_k \rangle}$$

Goal: Find $\beta_1, \ldots, \beta_k$ s.t. $\vec{z}$ is a representation of $\beta_1 \vec{y}_1 + \ldots + \beta_k \vec{y}_k$

~~$\vec{z}$ represents~~: $\vec{z} = \left\langle \alpha_1 w_1 \vec{y}_1 + \alpha_2 w_2 \vec{y}_2 + \ldots \alpha_k w_k \vec{y}_k, \underbrace{\sum_{j=1}^{k} \alpha_j w_j}_{} \right\rangle$

So $\vec{z}$ represents

$$\frac{\alpha_1 w_1}{\sum_j \alpha_j w_j}\vec{y}_1 + \quad \cdots \quad + \frac{\alpha_k w_k}{\sum_j \alpha_j w_j}\vec{y}_k$$

So $\beta_i = \dfrac{\alpha_i w_i}{\sum_j \alpha_j w_k}$ gives $\vec{z}$ represents $\sum_i \beta_i \vec{y}_i$

<u>Bottom line</u> $\quad \beta_i \propto \alpha_i w_i$

Conversely $\quad \alpha_i \propto \beta_i / w_i$

Hyperbolic interpolation

$$\mathrm{lerp}(\cancel{x}\vec{u}_1, \vec{u}_2, \beta) = \overbrace{(1-\beta)}^{\beta_1}\vec{u}_1 + \overbrace{\beta}^{\beta_2} u_2$$

Choose α s.t. $1-\alpha \propto 1-\beta$ $\qquad \alpha_1 = 1-\alpha$

$\alpha \propto \beta$ $\qquad \alpha_2 = \alpha$

Could help w/ Part 3 of the Quiz (Quiz 14)

Derivative of lerp-ing

$$\vec{u}(\alpha) = \mathrm{lerp}(\vec{x},\vec{y},\alpha) = (1-\alpha)\vec{x} + \alpha\vec{y}$$

Derivative w.r.t. α

$$\frac{d\vec{u}}{d\alpha} = -\vec{x}+\vec{y} = \vec{y}-\vec{x}$$

Does not depend on α.