Math 155A - Fall 2022 - Quiz #8 - October 20

Name: Answer Key

PID:

1. Recall that $\mathbf{k} = \langle 0, 0, 1\rangle$. Also, $\mathbf{k}^{\mathrm{T}}$ is the transpose of $\mathbf{k}$.
Give explicitly the 3×3 matrices that represent the following four linear maps on $\mathbb{R}^3$.

(a) $\mathbf{x} \mapsto \mathrm{Proj}_{\mathbf{k}}(\mathbf{x}) = (\mathbf{k}\,\mathbf{k}^{\mathrm{T}})\mathbf{x}$. (The projection of $\mathbf{x}$ onto $\mathbf{k}$.)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) $\mathbf{x} \mapsto \mathbf{x} - (\mathbf{k}\,\mathbf{k}^{T})\mathbf{x}$. (The component of of $\mathbf{x}$ perpendicular to $\mathbf{k}$.)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(c) $\mathbf{x} \mapsto \mathbf{x} \times \mathbf{k}$.

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(d) $\mathbf{x} \mapsto \mathbf{k} \times (\mathbf{x} \times \mathbf{k})$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Yes, this the same answer as for (b). Works for any unit vector, not just $\vec{k}$.