Math 155A - Fall 2022 - Quiz #9 - October 25

Name:

PID:

1. A light source is placed at $\langle -9, 0, 0\rangle$ and it casts shadows onto the plane P defined by $x = 3$. The $x = 3$ plane is parallel to the yz-plane and acts like an infinite wall.

When $\langle x, y, z\rangle$ is a point in $\mathbb{R}^3$ with $-9<x\leq 3$, define $A(\langle x, y, z\rangle)$ to be the position of the shadow of the point on the yz-plane. For example, $A(\langle -3, 2, 1\rangle) = \langle 3, 4, 2\rangle$, and $A(\langle -6, 2, 1\rangle) = \langle 3, 6, 3\rangle$.

(a) Working in ordinary coordinates (not homogeneous) give the formula expressing the mapping $A(\langle x, y, z\rangle) = \langle x', y', z'\rangle$. That is, give formulas for x', y', z' in terms of x, y, z.

$$x' = 3$$

$$y' = \frac{12y}{x+9}$$

$$z' = \frac{12z}{x+9}$$

$$\langle x, y, z\rangle \mapsto \left\langle 3, \frac{12\cdot y}{x+9}, \frac{12\cdot z}{x+9}\right\rangle$$

(b) Give a 4×4-matrix that represents the transformation A over homogeneous coordinates.

$$\langle x, y, z, 1\rangle \mapsto \langle 3x+27, 12\cdot y, 12\cdot z, x+9\rangle$$

$$\begin{pmatrix} 3 & 0 & 0 & 27 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 1 & 0 & 0 & 9 \end{pmatrix}$$