Math 155A - Fall 2022 - Quiz #11 - November 1

Name:

PID:

1. Let f be the linear transformation of $\mathbb{R}^2$ defined by

$$f(\langle x,y\rangle) = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Suppose $\mathbf{p} = \langle 0,3\rangle$ and that $\langle 2,-1\rangle$ is normal to $\mathcal{C}$ at $\mathbf{p}$. Give a vector $\mathbf{m}$ that is normal to the point $f(\mathbf{p})$ on the transformed curve $f(\mathcal{C})$. (It does not need to be a unit vector.)

$$(M^{-1})^T = \begin{pmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{pmatrix}$$

$$\vec{m} = \begin{pmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 7/6 \\ -1/3 \end{pmatrix}$$

2. Now let g be the affine transformation of $\mathbb{R}^2$ defined by

$$f(\langle x,y\rangle) = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

(So g is the composition of f and a translation.) Again suppose $\mathbf{p} = \langle 0,3\rangle$ lies on a curve $\mathcal{C}$ and that $\langle 2,-1\rangle$ is normal to $\mathcal{C}$ at $\mathbf{p}$. Give a vector $\mathbf{m}$ that is normal to the point $g(\mathbf{p})$ on the transformed curve $f(\mathcal{C})$.

Translations do not affect normals

$$\vec{m} = \begin{pmatrix} 7/6 \\ -1/3 \end{pmatrix}$$