Math 155A - Fall 2022 - Quiz #15 - November 22

Name: Answer Key

PID:

1. Let $\mathbf{x} = \langle -2, 0, 4\rangle$ and $\mathbf{y} = \langle 4, 6, -2\rangle$ be points in $\mathbb{R}^3$. Let $\mathbf{u} = \langle -4, 0, 8, 2\rangle$ and $\mathbf{v} = \langle 12, 18, -6, 3\rangle$ be homogeneous representions of $\mathbf{x}$ and $\mathbf{y}$ (respectively). Find scalars α and β so that $\alpha + \beta = 1$ and so that $\mathbf{w} = \alpha\mathbf{u} + \beta\mathbf{v}$ is a homogeneous representation of the midpoint of $\mathbf{x}$ and $\mathbf{y}$. In other words, $\mathbf{w}$ is an affine combination of $\mathbf{u}$ and $\mathbf{v}$, and a homogeneous representation of $\frac{1}{2}(\mathbf{x} + \mathbf{y})$.

$\alpha = \frac{3}{5}$, $\beta = \frac{2}{5}$

(To solve this, just pay attention to the weights 2 and 3 of $\vec{u}$ and $\vec{v}$. Need $\alpha \propto \frac{1}{3}$, $\beta \propto \frac{1}{3}$ and $\alpha + \beta = 1$.)

"$\propto$" means "proportional to"

2. A triangle in $\mathbb{R}^2$ has three vertices $\mathbf{x} = \langle 0, 0\rangle$, $\mathbf{y} = \langle 3, 3\rangle$ and $\mathbf{z} = \langle 6, 0\rangle$. The point $\mathbf{a} = \langle 3, 1\rangle$ has barycentric coordinates $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$, and $\gamma = \frac{1}{3}$.

Let $\mathbf{u} = \langle 2\mathbf{x}; 2\rangle$, $\mathbf{v} = \langle \mathbf{y}; 1\rangle$ and $\mathbf{w} = \langle 2\mathbf{z}; 2\rangle$ be homogeneous representations of $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$, respectively.

Find values α', β', γ' so that the affine combination $\alpha'\mathbf{u} + \beta'\mathbf{v} + \gamma'\mathbf{w}$ is a homogeneous representation of $\mathbf{u}$.

$\alpha' = \frac{1}{4}$, $\beta' = \frac{2}{4}$, $\gamma' = \frac{1}{4}$.

This was easy to solve since $\alpha = \beta = \gamma = 1/3$.

Need $\alpha' = \dfrac{\alpha/w_x}{\alpha/w_x + \beta/w_y + \gamma/w_z}$ and similarly for β', γ'

where w_x, w_y, w_z are the wieghts of w_x, w_y, w_z

Alternatively: $\alpha' \propto \alpha/w_x$, $\beta' \propto \beta/w_y$, $\gamma' \propto \gamma/w_z$.