

Math 154, Winter 2019

Homework 3

Due: Monday, January 28 by 5PM in basement of AP&M

(1) Consider the equation

$$x_1 + \cdots + x_6 = 32.$$

For each of the following conditions, how many solutions are there? (Each part is an independent problem, don't combine the conditions.) Explain your answer in each case.

- (a) The x_i are non-negative even integers.
- (b) The x_i are non-negative odd integers.
- (c) The x_i are positive integers.
- (d) The x_i are non-negative integers and $x_6 \leq 2$.

(2) How many positive integers ≤ 42000 are not divisible by any of 2, 3, 7?

(3) How many ways are there to list the letters of the word MATHEMATICS so that no two consecutive letters are the same?

(4) Let r, m, n be positive integers. Prove that

$$\sum_{i=0}^r \binom{m}{i} \binom{n}{r-i} = \binom{m+n}{r}.$$

(5) Let $n \geq 2$ be an integer.

(a) Prove that

$$\sum_{i=0}^n i \binom{n}{i} (-1)^{i-1} = 0.$$

(b) Deduce from (a) that

$$\sum_{\substack{0 \leq i \leq n \\ i \text{ odd}}} i \binom{n}{i} = \sum_{\substack{0 \leq i \leq n \\ i \text{ even}}} i \binom{n}{i}$$

and compute the common value.