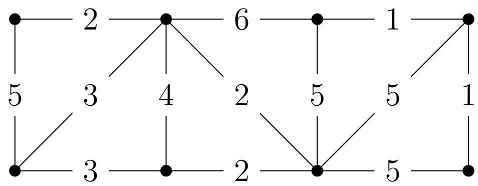
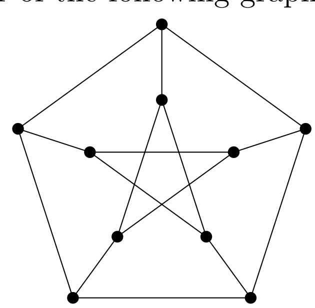
Math 154, Winter 2019
Homework 6
Due: Monday, Feb. 25 by 5PM in basement of AP&M

(1) Let $n \geq 3$ be an integer. Let K_n be the complete graph with vertices $1, \dots, n$ and let $G = K_n \setminus \{n, n-1\}$ and $H = K_n / \{n, n-1\}$. Compute the number of spanning trees of G and also of H.

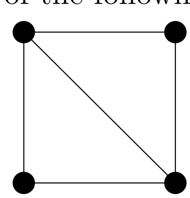
(2) (a) Given a graph with weights on its edges, describe an algorithm to find a *maximum* weight spanning tree, i.e., a spanning tree whose weight is as big as possible. Also prove that your method is correct.
(b) Find a minimum weight spanning tree in the following graph. Also, use your algorithm from (a) to find a maximum weight spanning tree. What are their weights?



(3) What is the chromatic number of the following graph (with 10 vertices)? Explain.



(4) What is the chromatic polynomial of the following graph? Show work or explain.



(5) Let G be a graph without loops. Let n be the number of vertices and let $\chi_G(z)$ be its chromatic polynomial. Recall from HW5 that $\Omega(G)$ is the number of cycle-free orientations of G. Show that

$$\Omega(G) = (-1)^n \chi_G(-1).$$

Hints

2a: Your proof should be fairly short and use Kruskal's greedy algorithm.

5: Do induction on the number of edges and use that both $\Omega(G)$ and $(-1)^n\chi_G(-1)$ satisfy similar kinds of deletion-contraction recursions.